Including tetraquark operators in the low-lying scalar meson sectors in lattice QCD

Danny Darvish

Ruairí Brett, Jacob Fallica, Andrew Hanlon, Colin Morningstar

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Carnegie Mellon University

Motivation

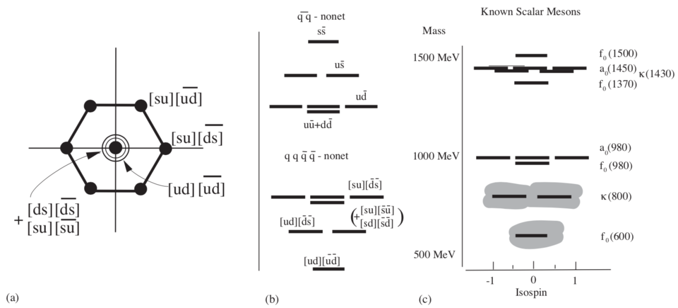
- Determine if addition of tetraquark operators yields additional levels in our lattice QCD spectrum

- Investigate the possible tetraquark content of the light scalar mesons

Tetraquark candidates in $\mathrm{N_f} = 2+1$ QCD

Several light scalar meson resonances have been proposed to have tetraquark content, such as the

- $a_0(980)$
- $a_0(1450)$
- $K_0^*(700)$
- $K_0^*(1430)$
- ...



Jaffe, R.L. Phys.Rept. 409 (2005) 1-45 hep-ph/0409065

We focus here on the isodoublet $K_0^*(700)$ (κ) and the isovector $a_0(980)$ resonances.

Spectroscopy in lattice QCD

In lattice QCD, we work in Euclidean spacetime ($t \to -i\tau$). We extract energies from correlation functions:

$$\langle 0|\, T\ \mathcal{O}(t+t_0)\overline{\mathcal{O}}(t_0)\,|0\rangle = \sum_n \langle 0|\,\mathcal{O}(0)\,|n\rangle\,\langle n|\,\overline{\mathcal{O}}(0)\,|0\rangle\, e^{-E_n t},$$

$E_0 \equiv 0$.

$$\langle \mathcal{O} \rangle = \frac{1}{Z}\int \mathcal{D}[\overline{\psi},\psi,U]\mathcal{O}e^{-S[\overline{\psi},\psi,U]}$$

In order to extract E_n, $\overline{\mathcal{O}}\,|0\rangle$ must have nonvanishing overlap with $|n\rangle$.

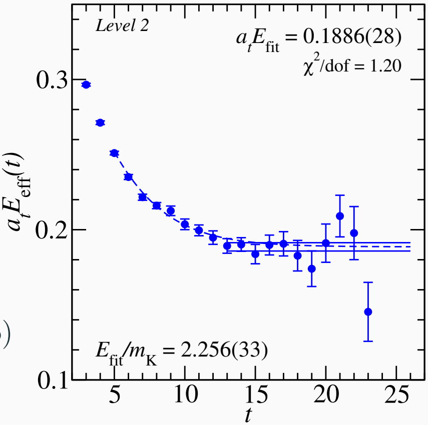
Spectroscopy in lattice QCD

$$C(t) = \sum_n \langle 0| \mathcal{O}(0) |n\rangle \langle n| \overline{\mathcal{O}}(0) |0\rangle e^{-E_n t}$$

$$E_{\text{eff}}(t) \equiv -\frac{1}{\Delta t} \ln\left(\frac{C(t+\Delta t)}{C(t)}\right).$$

$$\lim_{t\to\infty} E_{\text{eff}}(t) = E_0$$

Finite statistics →
can only extract lowest (non-zero) energy in $C(t)$.



Spectroscopy in lattice QCD

Want more than just the lowest level!

$$C_{ij}(t) \equiv \langle 0| T\ \mathcal{O}_i(t+t_0)\overline{\mathcal{O}}_j(t)\,|0\rangle$$

$$\begin{aligned} C_{ij}(t) &= \sum_n \langle 0|\,\mathcal{O}_i(0)\,|n\rangle\,\langle n|\,\overline{\mathcal{O}}_j(0)\,|0\rangle\, e^{-E_n t} \\ &= \sum_n Z_i^{(n)} Z_j^{(n)*} e^{-E_n t} \end{aligned}$$

Theorem. *For every $t \geq 0$, let $\lambda_n(t)$ be the eigenvalues of an $N \times N$ Hermitian correlation matrix $C(t)$, ordered such that $\lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_{N-1}$. Then,*

$$\lim_{t\to\infty} \lambda_n(t) = b_n e^{-E_n t}\left(1 + \mathcal{O}\left(e^{-t\Delta_n}\right)\right), \quad b_n > 0, \quad \Delta_n = \min_{m\neq n} |E_n - E_m|\,.$$

Spectroscopy in lattice QCD

$$\lim_{t\to\infty}\lambda_n(t) = b_n e^{-E_n t}\left(1+\mathcal{O}\left(e^{-t\Delta_n}\right)\right)$$

Signal-to-noise problem: when $\mathcal{O}(e^{-t\Delta_n})$ is small, $C(t)$ is poorly determined.

Solution: GEVP

$$C(t)v_n(t,\tau_0) = \lambda_n(t,\tau_0)\,C(\tau_0)\,v_n(t,\tau_0), \qquad t>\tau_0$$

$$\lambda_n^{(0)}(t,\tau_0) = e^{-E_n(t-\tau_0)}\left(1+\mathcal{O}\left(e^{-(E_N-E_n)t}\right)\right)$$

Operator construction

A single meson operator looks like:

$$\Phi^{AB}_{\alpha\beta;ijk}(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}\cdot\left(\boldsymbol{x}+\frac{1}{2}\left(\boldsymbol{d}_\alpha+\boldsymbol{d}_\beta\right)\right)}\delta_{ab}\overline{q}^{A}_{a\alpha i}(\boldsymbol{x},t)q^{B}_{b\beta jk}(\boldsymbol{x},t)$$

Displacements:

Operator construction

CG series for four SU(3) color objects:

$$3 \otimes 3 \otimes 3 \otimes 3 = 3 \oplus 3 \oplus 3 \oplus \overline{6} \oplus \overline{6} \oplus 15 \oplus 15 \oplus 15 \oplus 15,$$
$$3 \otimes 3 \otimes 3 \otimes \overline{3} = \overline{3} \oplus \overline{3} \oplus \overline{3} \oplus 6 \oplus 6 \oplus 6 \oplus \overline{15} \oplus \overline{15} \oplus 24,$$
$$3 \otimes 3 \otimes \overline{3} \otimes \overline{3} = 1 \oplus 1 \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus \overline{10} \oplus \overline{10} \oplus 27.$$

- Only one of these combinations gives us color singlet states.

Operator construction

$$3 \otimes 3 \otimes \overline{3} \otimes \overline{3} = 1 \oplus 1 \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus \overline{10} \oplus \overline{10} \oplus 27$$

There are two color singlets to consider. 81 color basis vectors, p_a, q_b, r_c, s_d, where $p_a, q_b, \ldots$ transform in 3 rep. and $p_a^*, q_b^* \ldots$ transform in the $\overline{3}$ rep.

Two gauge-invariant linearly independent combinations:

$$T_S = (\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})\, p_a^*(x) q_b^*(x) r_c(x) s_d(x)$$
$$T_A = (\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc})\, p_a^*(x) q_b^*(x) r_c(x) s_d(x)$$

Operator construction

Definite-momentum operator:

$$\Phi^{ABCD\pm}_{\alpha\beta\mu\nu;ijkl}(t) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \left(\delta_{ab}\delta_{cd} \pm \delta_{ad}\delta_{bc}\right) \overline{q}^{A}_{a\alpha i}(\boldsymbol{x},t) q^{B}_{b\beta j}(\boldsymbol{x},t) \overline{q}^{C}_{c\mu k}(\boldsymbol{x},t) q^{D}_{d\nu l}(\boldsymbol{x},t)$$

Tetraquark operator projects entire $\Phi^{ABCD\pm}_{\alpha\beta\mu\nu;ijkl}$ on to relevant symmetry channels, l:

$$T_{l\pm}(t) = c^{(l)}_{\alpha\beta\mu\nu;ijkl} \Phi^{ABCD\pm}_{\alpha\beta\mu\nu;ijkl}(t)$$

Cf. meson operator, which projects individual $q\bar{q}$ objects onto relevant symmetry channels.

Lattice runs

$32^3 \times 256$ lattice, 412 configs, $\frac{a_s}{a_t} \approx 3.451$, $L = 3.74$ fm,
$m_\pi \approx 230$ MeV, $m_K \approx m_{K,\text{phys}}$

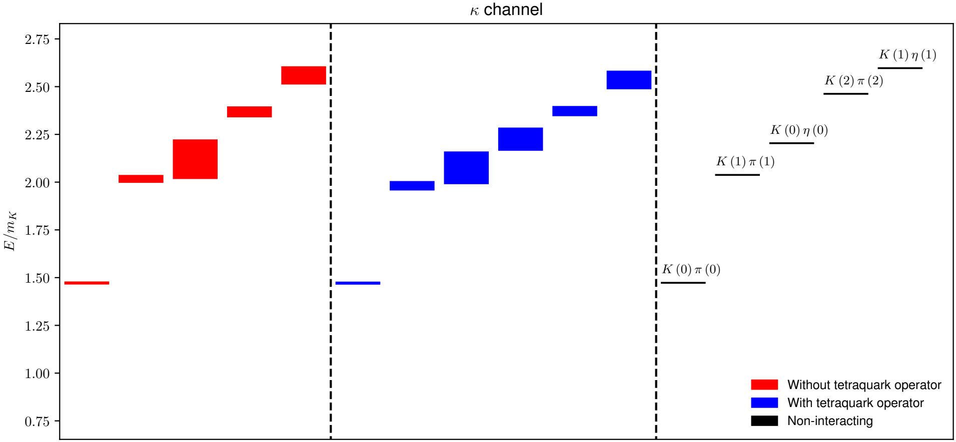
Quark propagation treated using stochastic LapH method

Operator basis containing single and two-meson interpolating operators, as well as additional tetraquark operators.

Operator bases

κ channel	a_0 channel
$\bar{s}u(0)^{TDU5}_{A1_g}$	$u\bar{d}(0)^{SD2}_{A1_g^-}$
$\bar{s}u\bar{s}s(0)^{SS7}_{A1_g}$	$\bar{u}u\bar{d}u(0)^{SS3}_{A1_g^-}$
$K(0)^{SS0}_{A1_u}\ \pi(0)^{SS0}_{A1_u^-}$	$K(0)^{SS0}_{A1_u}\ \bar{K}(0)^{SS0}_{A1_u}$
$K(0)^{SS0}_{A1_u}\ \eta(0)^{SS0}_{A1_u^+}$	$K(1)^{SS1}_{A2}\ \bar{K}(1)^{SS1}_{A2}$
$K(0)^{SS0}_{A1_u}\ \phi(0)^{SS0}_{A1_u^+}$	$\eta(0)^{SS0}_{T1_u^-}\ \pi(0)^{SS0}_{T1_u^+}$
$K(1)^{SS1}_{A2}\ \pi(1)^{SS1}_{A2^-}$	$\eta(1)^{SS0}_{A2^+}\ \pi(1)^{SS0}_{A2^-}$
$K(1)^{SS1}_{A2}\ \eta(1)^{SS1}_{A2^+}$	$\eta(2)^{SS1}_{A2^+}\ \pi(2)^{SS1}_{A2^-}$
$K(1)^{SS1}_{A2}\ \phi(1)^{SS1}_{A2^+}$	$\phi(0)^{SS0}_{A1_u^+}\ \pi(0)^{SS0}_{A1_u^-}$
$K(2)^{SS0}_{A2}\ \pi(2)^{SS0}_{A2^-}$	$\phi(0)^{SS0}_{T1_u^-}\ \pi(0)^{SS0}_{T1_u^+}$
$K(3)^{SS0}_{A2}\ \pi(3)^{SS0}_{A2^-}$	$\phi(1)^{SS1}_{A2^+}\ \pi(1)^{SS1}_{A2^-}$
	$\phi(2)^{SS0}_{A2^+}\ \pi(2)^{SS0}_{A2^-}$

Lattice spectrum in the $\kappa/K_0^*(700)$ sector (preliminary)



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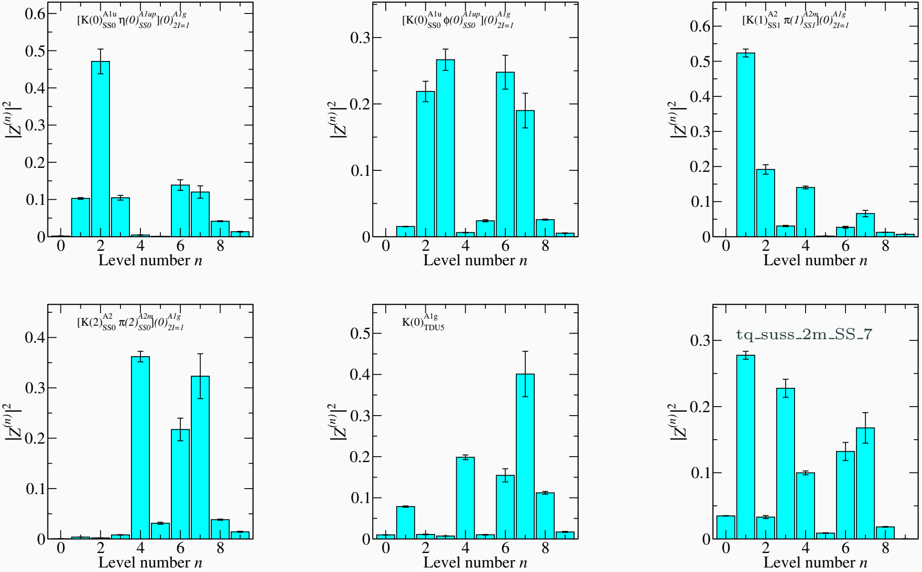
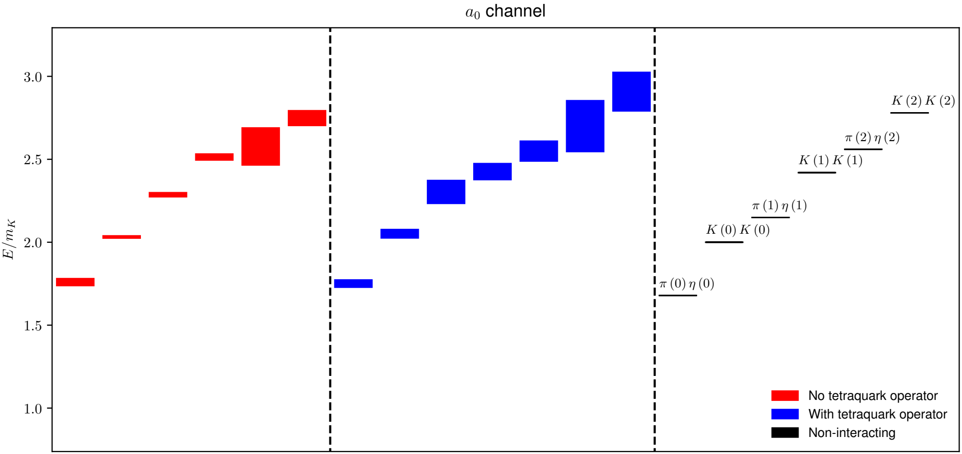
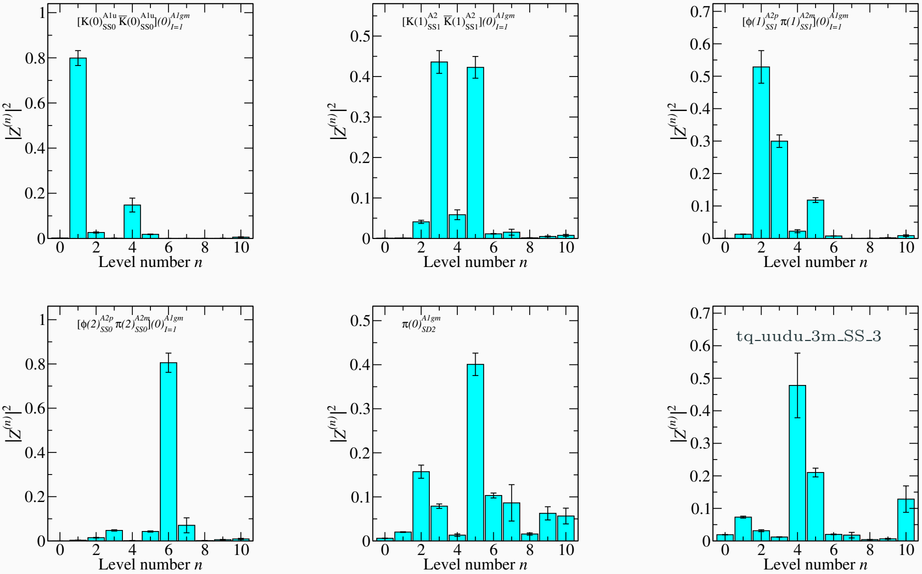


Lattice spectrum in the $a_0(980)$ sector (preliminary)



Lattice spectrum in the $a_0(980)$ sector (preliminary)



Conclusions and future work

- It has been proposed that the scalar resonances κ and $a_0(980)$ could have tetraquark content
- Including tetraquark interpolating operators in the symmetry channels where we would see these resonances yields an extra state
- First time to do this in $N_f = 2 + 1$ without neglecting disconnected contributions (for κ)
- Lüscher analysis to obtain scattering amplitudes
- Three-particle scattering formalism in progress (see M. Mai's talk Monday)