Single-spin asymmetry at subleading level

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Single transverse spin asymmetry (SSA)

Consider a transversely polarized proton scatter off an unpolarized proton or electron

\[ A_N \equiv \frac{L - R}{L + R} = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow} \]

\[ x_F \sim \frac{2P_z}{\sqrt{s}} \]

discaled longitudinal momentum
Mechanism

• There exists correlation proportional to

\[ \varepsilon_{\mu\nu\rho\lambda} S^\mu_T p^\nu_{hT} \cdots \]

• To generate such term in Feynman diagram, need

\[ tr[\gamma_5 S_T p_{hT} \cdots] = i \varepsilon_{\mu\nu\rho\lambda} S^\mu_T p^\nu_{hT} \cdots \]

• Projector for polarized proton \((p + m)\gamma_5 S_T\)
• Projector for produced hadron \(p_h + m_h\)
• But need strong phase to make cross section real
Where is phase?

• Phase comes from on-shell internal particles:

\[
\frac{1}{k^2 + i\epsilon} = \frac{P}{k^2} - i\pi\delta(k^2)
\]

• Need time-like final states with FSI
• No phase at LO and one loop
Phase at two loops

- Need two final-state particles with one gluon exchange (FSI) between them.
- Nonvanishing phase appears at two loops, and comes from box diagram.
Kinematics for phase

\[ q = p_2 - p_1, \quad p_2 = (p_2^+, p_2^-, 0_T) \quad p_1^+, p_2^- \gg p_2^+ \gg \Lambda_{QCD} \]

\[ p_2^2 > 0 \quad \text{time-like} \]

\[ p_1 = (p_1^+, 0, 0_T) \]

\[ l_2^- = l_{2T}^2/(2l_2^+) \quad l_1^+ = \frac{1}{2} \left( p_2^+ \pm \sqrt{p_2^{+2} - 2\frac{p_2^+}{p_2^-}l_{1T}^2} \right) \]

\[ l_2^+ = \frac{1}{2} \left( p_2^+ \pm \sqrt{p_2^{+2} - 2\frac{p_2^+}{p_2^-}l_{2T}^2} \right) \]
Collinear to initial state

- Picking up plus signs, gluons collimate to polarized proton

\[ l_{1,2}^+ \sim O(p_2^+) \gg l_{1T,2T} \gg l_{1,2}^- \]
\[ p_1 - l_2 \approx p_1^+ - p_2^+ \]
\[ p_2 - l_1 \approx p_2^+ - l_2 \approx p_2^- \]

- Phase goes into Sivers function
- FSI gluon is soft
Sivers function

- Eikonalize outgoing quark and insert Fierz identity

\[ I_{ij} I_{lk} = \frac{1}{4} I_{ik} I_{lj} + \frac{1}{4} (\gamma^\alpha)_{ik} (\gamma_\alpha)_{lj} \]
\[ + \frac{1}{4} (\gamma^5 \gamma^\alpha)_{ik} (\gamma_\alpha \gamma^5)_{lj} + \frac{1}{4} (\gamma^5)_{ik} (\gamma^5)_{lj} \]
\[ + \frac{1}{8} (\gamma^5 \sigma^{\alpha\beta})_{ik} (\sigma_{\alpha\beta} \gamma^5)_{lj} \]

give dominant (twist-2) contribution

pick up \( p = p^+ \gamma^- \)
Parton transverse momentum

- Sivers function demands inclusion of parton transverse momentum

\[ \text{compensated by phase here} \]

\[ tr \left[ \gamma_5 S_T k_T \gamma^+ \gamma^- \cdots \right] = i \varepsilon_{\mu \nu} S_T^\mu k_T^\nu \cdots \]

- This correlation determines preferred direction of \( k_T \) for polarized proton, which then propagates into \( p_h \)
Spin-transverse-momentum correlation

\[ f^{q/p\uparrow}_{\perp T}(x, k_T, \vec{S}_T) = f_{q/p}(x, k_T) - \frac{1}{M} f^{1T}_{1T}(x, k_T) \vec{S}_T \cdot (\hat{p}_h \times k_T) \]

produced hadron tends to move to right
Collinear to final state

- Picking up minus signs, gluons collimate to produced hadron

\[ l_{1,2}^- \sim O(p_2^-) \gg l_{1T,2T} \gg l_{1,2}^+ \]

\[ p_2 - l_1 \sim O(p_2^-), \quad p_2 - l_2 \sim O(p_2^-) \]

\[ p_1 - l_2 \text{ highly off-shell} \]

- Phase goes into Collins fragmentation function

Collins 1993
Collins function

• Eikonalize incoming quark and insert Fierz identity
  \( \gamma_5 \sigma^{-y} \) dominates

• Collins function demands inclusion of parton \( k_T \)

• LO hard kernel demands projector for initial state

• Transversity distribution defined

\( \mathcal{P}_h = p_h \gamma^+ \)

\( \gamma_5 \sigma^{-y} \) polarized quark

\( \gamma_5 \sigma^{+y} \) hard kernel

\( (p + m)\gamma_5 S_T \)
Mechanism

- Transversity distribution for polarized proton determines preferred direction of quark spin
- This polarized quark scattered into final state
- Collins function then determines direction of polarized quark (produced hadron) preferred by correlation
- Without preferred direction of quark spin from initial state, Collins function cannot work
Twist-2 TMDs

\[
\Phi[\gamma^+] = f_1 - \frac{\epsilon_T^{\rho\sigma} p_T \rho S_T \sigma}{M} f_{1T},
\]

\[
\Phi[\gamma^+\gamma_5] = S_L g_{1L} - \frac{p_T \cdot S_T}{M} g_{1T},
\]

\[
\Phi[i\sigma^\alpha+\gamma_5] = S_T^\alpha h_{1L} + S_L \frac{p_T^\alpha}{M} h_{1L}^\perp
\]

- \[
- \frac{p_T^\alpha p_T^{\rho}}{M^2} - \frac{1}{2} p_T^2 g_T^{\alpha\rho} S_T \rho h_{1T}^\perp - \frac{\epsilon_T^{\alpha\rho} p_T \rho}{M} h_{1L}^\perp
\]

Boer, Mulders 1997
Goeke, Meta, Schlegel 2005
Bacchetta et al., 2007

Sivers function
in the case of FFs, it is Collins function
transversity function
Phase in hard kernel

• For other sign combinations, or arbitrary transverse momenta
• phase appears in hard kernel

\[ H^{(2)} = \]

• How to extract this phase?
• Use \( \gamma_5 \gamma^\perp \)
• A new contribution to SSA
2-parton twist-3 TMDs

$$\Phi[\gamma_5] = \frac{M}{P^+} \left[ S_L e_L - \frac{p_T \cdot S_T}{M} e_T \right], \quad \Phi[1] = \frac{M}{P^+} \left[ e - \frac{\epsilon_T^{\rho\sigma} p_T \rho S_T \sigma}{M} e_T \right]$$

$$\Phi[\gamma^\alpha] = \frac{M}{P^+} \left[ - \epsilon_T^{\alpha\rho} S_T \rho f_T - S_L \epsilon_T^{\alpha\rho} p_T \rho f_L \right. \right.$$

$$\left. - \frac{p_T^{\alpha} p_T^{\rho} - \frac{1}{2} p_T^2 g_T^{\alpha\rho}}{M^2} \epsilon_T^{\rho\sigma} S_T^{\sigma} f_T \frac{1}{M} + \frac{p_T^{\alpha}}{M} f_T \right]$$

$$\Phi[\gamma^\alpha \gamma_5] = \frac{M}{P^+} \left[ S_T^{\alpha} g_T + S_L \frac{p_T^{\alpha}}{M} g_L \right. \right.$$

$$\left. - \frac{p_T^{\alpha} p_T^{\rho} - \frac{1}{2} p_T^2 g_T^{\alpha\rho}}{M^2} S_T \rho g_T \frac{1}{M} - \epsilon_T^{\alpha\rho} p_T \rho g_T \right]$$

$$\Phi[i\sigma^{\alpha\beta} \gamma_5] = \frac{M}{P^+} \left[ \frac{S_T^{\alpha} p_T^{\beta}}{M} - \frac{p_T^{\alpha} S_T^{\beta}}{M} h_T \frac{1}{M} - \epsilon_T^{\alpha\beta} h \right], \right.$$

$$\Phi[i\sigma^{+-} \gamma_5] = \frac{M}{P^+} \left[ S_L h_L - \frac{p_T \cdot S_T}{M} h_T \right], \right.$$

Boer, Mulders 1997
Goeke, Meta, and Schlegel 2005
Bacchetta et al., 2007
Factorization of new contribution

• Unpolarized twist-2 fragmentation function

• Polarized quark scattered into direction preferred by correlation

\[ \text{tr}[\gamma_5\gamma^y p_{hT}\gamma^+\gamma^- \cdots] = i\epsilon \gamma^x_{yx+} p_{hT}^x \cdots \]

• 2-parton twist-3 TMD \( g_T \) defined for polarized proton

\[ p_h = p_h^+\gamma^+ \]

\[ \gamma^- \]

\[ \gamma^+ \]

\[ \gamma_5\gamma^y \]

\[ (p + m)\gamma_5 s_T \]
Lesson learned

- Both Sivers and Collins functions contribute starting from LO hard kernel
- If allowed to go to higher orders of hard kernel, other projectors can be used
- Though higher orders, data with $Q \sim \text{few GeV}$ (such as COMPASS), hard kernel effect may be sizable
- Hard kernel is process-dependent
  - Rich phenomenology!
At 3 loops

• At 3 loops, we can have 2-loop TMD for polarized proton and 1-loop hard kernel

• In addition to Sivers function, can use $\gamma^x$ to extract phase in initial state in this case

• 2-parton twist-3 TMD $f_T$ defined

• Another new contribution
2-parton twist-3 TMDs

\[ \Phi[i\gamma_5] = \frac{M}{P^+} \left[ S_L e_L - \frac{p_T \cdot S_T}{M} e_T \right] , \quad \Phi[1] = \frac{M}{P^+} \left[ e - \frac{\epsilon_T^{\rho\sigma} p_T \rho S_T \sigma}{M} e_T \right] \]

\[ \Phi[\gamma^\alpha] = \frac{M}{P^+} \left[ -\epsilon_T^{\alpha\rho} S_T \rho f_T - S_L \frac{\epsilon_T^{\alpha\rho} p_T \rho}{M} f_L \right. \]
\[ \left. - \frac{p_T^\alpha p_T^\rho - \frac{1}{2} p_T^2 g_T^{\alpha\rho}}{M^2} \epsilon_T \rho \sigma S_T^\sigma f_T + \frac{p_T^\alpha}{M} f_L \right] \]

\[ \Phi[\gamma^\alpha \gamma_5] = \frac{M}{P^+} \left[ S_T^\alpha g_T + S_L \frac{p_T^\alpha}{M} g_L \right. \]
\[ \left. - \frac{p_T^\alpha p_T^\rho - \frac{1}{2} p_T^2 g_T^{\alpha\rho}}{M^2} S_T \rho g_T^{\perp} - \frac{\epsilon_T^{\rho\sigma} p_T \rho S_T^\sigma}{M} g_L \right] \]

\[ \Phi[i\sigma^{\alpha\beta} \gamma_5] = \frac{M}{P^+} \left[ S_T^\alpha p_T^\beta - \frac{p_T^\alpha S_T^\beta}{M} h_T^{\perp} - \epsilon_T^{\alpha\beta} h \right] , \]

\[ \Phi[i\sigma^{+-} \gamma_5] = \frac{M}{P^+} \left[ S_L h_L - \frac{p_T \cdot S_T}{M} h_T \right] , \]
Up to to twist-3 NNLO

- Up to 2-parton twist-3 in TMD and FF, 2-loop in hard kernel, SSA is given by

\[ d\sigma = f_{1T}^{\perp} \otimes H_{\gamma^{-},\gamma^{+}}^{(0)} \otimes D_{1} + f_{1T}^{\perp} \otimes H_{\gamma^{-},\gamma^{x}}^{(1)} \otimes D_{\perp} + f_{1T}^{\perp} \otimes H_{\gamma^{-},\gamma_{5}\gamma^{x}}^{(2)} \otimes G_{\perp} \\
+ g_{1T} \otimes H_{\gamma_{5}\gamma^{-},\gamma^{+}}^{(2)} \otimes D_{1} + g_{1T} \otimes H_{\gamma_{5}\gamma^{-},\gamma_{5}\gamma^{y}} \otimes G_{\perp} + g_{1T} \otimes H_{\gamma_{5}\gamma^{-},\gamma^{y}} \otimes D_{\perp} \\
+ h_{1} \otimes H_{\gamma_{5}\sigma^{y-},\gamma_{5}\sigma^{y+}} \otimes H_{\perp}^{(0)} + h_{1} \otimes H_{\gamma_{5}\sigma^{y-},\gamma_{5}\sigma^{y+x}} \otimes H^{*} + h_{1} \otimes H_{\gamma_{5}\sigma^{y-},I} \otimes E^{*} \\
+ e_{T} \otimes H_{\gamma_{5},\gamma_{5}\sigma^{y+}}^{(1)} \otimes H_{\perp}^{(1)} + e_{T} \otimes H_{I,\gamma_{5}\sigma^{y+}} \otimes H_{\perp}^{(2)} \\
+ f_{T} \otimes H_{\gamma^{y},\gamma^{+}} \otimes D_{1} + g_{T} \otimes H_{\gamma_{5}\gamma^{y},\gamma^{+}} \otimes D_{1} \\
+ h_{T} \otimes H_{\gamma_{5}\sigma^{y+x},\gamma_{5}\sigma^{y+}} \otimes H_{\perp}^{(1)} + h_{T} \otimes H_{\gamma_{5}\sigma^{-},\gamma_{5}\sigma^{y+}} \otimes H_{\perp}^{(1)} \]
Sign change of Sivers function

\[ \text{Sivers}_{DY} = -1 \times \text{Sivers}_{SIDIS} \]
Sign-mismatch problem

- No sign flip seen in $p^+p \rightarrow \pi + X$

Kang, Qiu, Vogelsang, Yuan 2011

expectation from SIDIS data (HERMES, COMPASS) under sign flip
result extracted from data (E704, STAR, PHENIX, BRAHMS)

correlation function for polarized proton, assumed to dominate

$T_F^q(x, x) = -\int d^2p_\perp \frac{p_\perp^2}{M} f_1^{1q}(x, p_\perp^2)|_{SIDIS}$

- Now there are other twist-3 contributions...
Sivers Asymmetry in Drell-Yan: Hint of Sign Change!

arXiv:1704.00488 [hep-ex]
Origin of SSA?

• Large twist-3 fragmentation function? Kanazawa, Koike, Metz, Pitonyak, 2014

• $k_T$ factorization does not apply to Drell-Yan

• Twist-3 FF implies $A_N \propto A^{-1/3}$ in pA collision? Hatta, Xiao, Yoshidac, Yuan 2016

STAR, DIS 2016 showed
Need to study complete subleading contributions to understand all data and identify origin of SSA!
Summary and outlook

• There are many sources to SSA
• Have derived complete two-parton twist-3 contributions to SSA up to two loops in hard kernel
• Only gT contribution survives in collinear factorization
• Rich phenomenology is expected, eg., impact on extraction of Sivers, Collins functions?
• Will study subleading contributions to understand all data and identify origin of SSA
Back-up slides
Twist-2 TMDs

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<thead>
<tr>
<th>Quark</th>
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<tbody>
<tr>
<td>Nucleon</td>
<td>number density $f_1^q(x, k_T^2)$</td>
<td>Boer-Mulders $h_1^{q1}(x, k_T^2)$</td>
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<td>spin of the nucleon</td>
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<td>Spin of the quark</td>
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<td>$k_T$ of the quark</td>
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$kT$ can be integrated out in transverse distribution
Twist-3 TMDs

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<tr>
<th>Twist-3 TMDs</th>
<th>Formula</th>
<th>Description</th>
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<td>$e(x, k_\perp)$,</td>
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<td>number density</td>
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<td>$f^\perp(x, k_\perp)$</td>
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<td>Sivers function</td>
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<td>$e_T^\perp(x, k_\perp)$</td>
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<td>helicity distribution</td>
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<td>$f_T^{1\perp}(x, k_\perp)$</td>
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<td>Worm gear: trans-helicity</td>
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3-parton twist-3

• Note the 3-parton twist-3 (Qiu-Sterman) contribution in, for example, Drell-Yan

\[ \phi_{i/A}^{(3)}(x, x') \otimes \phi_{j/B}^{(3)}(y, y') \otimes D_{h/c}(z) \otimes H_{ij\rightarrow c}^{(A)}(x, x', y, z) \]
\[ + \phi_{i/A}(x) \otimes \phi_{j/B}^{(3)}(y, y') \otimes D_{h/c}(z) \otimes H_{ij\rightarrow c}^{(B)}(x, y, y', z) \]
\[ + \phi_{i/A}(x) \otimes \phi_{j/B}(y) \otimes D_{h/c}^{(3)}(z, z') \otimes H_{ij\rightarrow c}^{(C)}(x, y, z, z') \]

• New contribution deserves study

Qiu, Sterman 1991
Kouvaris, Qiu, Vogelsang, Yuan 2006
Yuan, Zhou 2008
Kang, Qiu, Vogelsang, Yuan 2008