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# From $\bar{K}N$ interactions to $\bar{K}$ -nuclear quasi-bound states

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*acknowledging also input from E. Friedman and A. Gal, Hebrew University, Jerusalem*

# Outline

- ① Chirally motivated  $K^-N$  approaches
- ②  $K^-N$  treatment in nuclear medium
- ③ Kaonic atoms (and what we can learn from them)
- ④ Kaonic nuclei predictions
- ⑤ Modelling the in-medium  $K^-NN$  absorption
- ⑥ Summary

# Chirally motivated $K^- N$ approaches

$\bar{K}N - \pi\Sigma$  system (+  $\pi\Lambda$ ,  $\eta\Lambda$ ,  $\eta\Sigma$ ,  $K\Xi$ )

meson octet - baryon octet coupled channels interactions

adding the meson singlet for  $\eta N$ ,  $\eta' N$  - P. Bruns, A. C., arXiv:1903.10350[nucl-th]

- strongly interacting multichannel system with an s-wave resonance, the  $\Lambda(1405)$ , just below the  $K^- p$  threshold
- modern theoretical treatment based on **effective chiral Lagrangians**
- effective potentials constructed to match the chiral meson-baryon amplitudes up to LO or NLO order
- Lippmann-Schwinger (or Bethe-Salpeter) equation to sum the major part of the perturbation series
- **low energies around threshold** - **only s-wave considered in most approaches**

$$T = V + V G T$$



# Chirally motivated $K^-N$ approaches

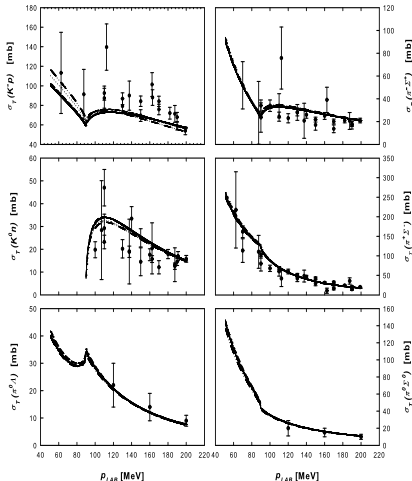
- Kyoto-Munich (KM)  
*Y. Ikeda, T. Hyodo, W. Weise, Nucl. Phys. A 881 (2012) 98*
- Murcia ( $M_I, M_{II}$ )  
*Z. H. Guo, J. A. Oller, Phys. Rev. C 87 (2013) 035202*
- Bonn ( $B_2, B_4$ )  
*M. Mai, U.-G. Meißner - Eur. Phys. J. A 51 (2015) 30*
- Prague (P)  
*A. C., J. Smejkal, Nucl. Phys. A 881 (2012) 115*
- Barcelona (BCN)  
*A. Feijoo, V. Magas, À. Ramos, Phys. Rev. C 99 (2019) 035211*

Model parameters (couplings, inverse interaction ranges or subtraction constants) fixed in **fits to low energy  $K^-p$  data** (and more in some cases).

Comparative analysis of the first four approaches presented in  
*A. C., M. Mai, U.-G. Meißner, J. Smejkal - Nucl. Phys. A 954 (2016) 17*

# $K^-p$ data (at and above threshold)

low energy cross sections:



threshold branching ratios:

$$\gamma = \frac{\Gamma(K^-p \rightarrow \pi^+\Sigma^-)}{\Gamma(K^-p \rightarrow \pi^-\Sigma^+)} = 2.36 \pm 0.04$$

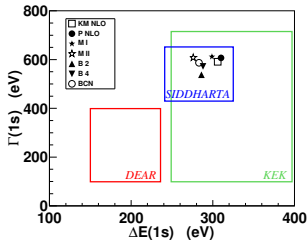
$$R_C = \frac{\Gamma(K^-p \rightarrow \text{charged})}{\Gamma(K^-p \rightarrow \text{all})} = 0.664 \pm 0.011$$

$$R_n = \frac{\Gamma(K^-p \rightarrow \pi^0\Lambda)}{\Gamma(K^-p \rightarrow \text{neutral})} = 0.189 \pm 0.015$$

kaonic hydrogen:

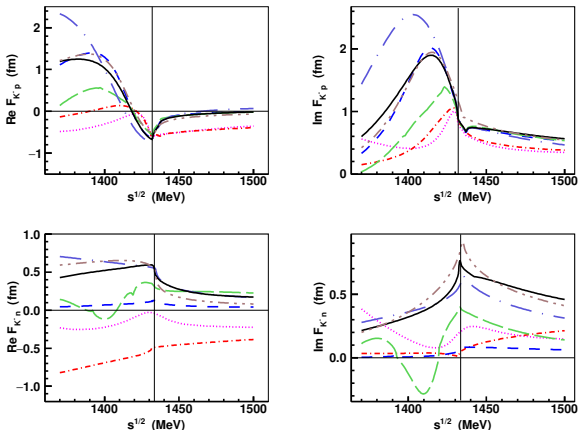
$$\Delta E_N(1s) = 283 \pm 36(\text{stat.}) \pm 6(\text{syst.}) \text{ eV}$$

$$\Gamma(1s) = 541 \pm 89(\text{stat.}) \pm 22(\text{syst.}) \text{ eV}$$



# Model predictions - $K^-N$ amplitudes (free space)

$K^-p$  and  $K^-n$  elastic amplitudes



$B_2$  (dotted, purple),  $B_4$  (dot-dashed, red),  $M_I$  (dashed, blue),  $M_{II}$  (long-dashed, green),  $P$  (dot-long-dashed, violet),  $BCN$  (dot-dot-dashed, brown),  $KM$  (continuous, black)

# In-medium $K^-N$ amplitudes

nuclear medium impact: Pauli (anti)correlations, hadron self-energies

- **WRW** method (based on multiple scattering theory)

*T. Wass, M. Rho, W. Weise, Nucl. Phys. A 617 (1997) 449*

$$F_1 = \frac{\frac{\sqrt{s}}{m_N} F_{K^-n}(\sqrt{s})}{1 + \frac{1}{4} \xi_k \frac{\sqrt{s}}{m_N} F_{K^-n}(\sqrt{s}) \rho}, \quad F_0 = \frac{\frac{\sqrt{s}}{m_N} [2F_{K^-p}(\sqrt{s}) - F_{K^-n}(\sqrt{s})]}{1 + \frac{1}{4} \xi_k \frac{\sqrt{s}}{m_N} [2F_{K^-p}(\sqrt{s}) - F_{K^-n}(\sqrt{s})] \rho}$$

where  $\xi_k = \frac{9\pi}{\rho_F^2} 4 \int_0^\infty \frac{dt}{t} \exp(iqt) j_1^2(t), \quad q = \frac{1}{\rho_F} \sqrt{\omega_{K^-}^2 - m_{K^-}^2}.$

- **P + Pauli + SE** model (Green function integral modified)

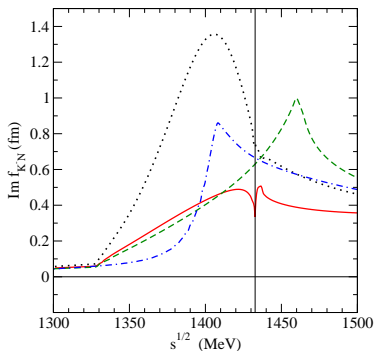
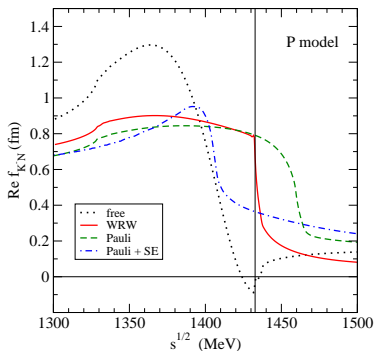
*A. C., E. Friedman, A. Gal, D. Gazda, J. Mareš, Phys. Rev. C 84 (2011) 045206*

*A. C., J. Smejkal, Nucl. Phys. A 881 (2012) 115*

$$F_{ij}(\sqrt{s}; \rho) = \left[ V^{-1}(\sqrt{s}) - G(\sqrt{s}; \rho) \right]_{ij}^{-1}$$

$$G_j(\sqrt{s}; \rho) = -4\pi \int_{\Omega_j(\rho)} \frac{d^3 p}{(2\pi)^3} \frac{g_j^2(p)}{p_j^2 - p^2 - \Pi_j(\sqrt{s}, \vec{p}; \rho) + i0}$$

## In-medium $K^-N$ amplitudes



Energy dependence of reduced free-space (dotted line)  $f_{K^-N} = \frac{1}{2}(f_{K^-p} + f_{K^-n})$  amplitude compared with **WRW modified** amplitude (solid line), **Pauli blocked** (dashed line), and **Pauli + SE** (dot-dashed line) modified amplitude for  $\rho_0 = 0.17 \text{ fm}^{-3}$  in the P model.



# $K^-$ -nuclear optical potential

coherent sum of  $\bar{K}N$  scatterings:  $V_{\text{opt}}^{(1)} \approx t_{\bar{K}N} \rho$

$$\Pi_{K^-} = 2\text{Re}(\omega_{K^-})V_{K^-}^{(1)} = -4\pi \left( F_0 \frac{1}{2}\rho_p + F_1 \left( \frac{1}{2}\rho_p + \rho_n \right) \right)$$

$F_0$  and  $F_1$  – isospin 0 and 1 in-medium  $K^-N$  amplitudes

amplitudes evaluated at energies shifted with respect to the two-body ones!

- the amplitudes are a function of  $\sqrt{s}$ ,  $s = (E_N + E_{K^-})^2 - (\vec{p}_N + \vec{p}_{K^-})^2$
- $K^-N$  c.m. frame  $\rightarrow K^-$ -nucleus frame, where  $\vec{p}_N + \vec{p}_{K^-} \neq 0$

$$\sqrt{s} = E_{th} - B_N \frac{\rho}{\bar{\rho}} - \frac{\mu_{KN}}{m_K} \left[ B_{K^-} \frac{\rho}{\rho_{max}} + 23 \left( \frac{\rho}{\bar{\rho}} \right)^{2/3} + V_C \left( \frac{\rho}{\rho_{max}} \right)^{1/3} \right] + \frac{\mu_{KN}}{m_N} \text{Re} V_{K^-}(r)$$

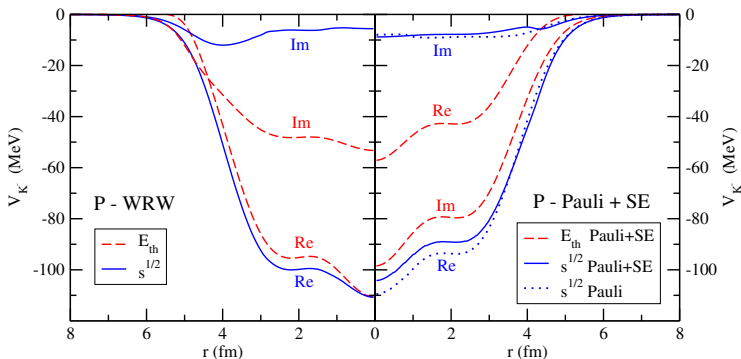
$B_N = 8.5$  MeV and  $\mu_{KN} = m_N m_K / (m_N + m_K)$

low-density limit  $\delta\sqrt{s} \rightarrow 0$  as  $\rho \rightarrow 0$ , where  $\delta\sqrt{s} = \sqrt{s} - E_{th}$ .

*A. C., E. Friedman, A. Gal, D. Gazda, J. Mareš, PLB 702 (2011) 402*

- $B_{K^-}$  and  $V_{K^-} \Rightarrow$  self-consistency scheme in  $\bar{K}$ -nuclear applications

# $K^-$ -nuclear optical potential



$K^-$  nuclear potential in  $^{40}\text{Ca}$  calculated using  $K^-N$  P amplitudes at threshold (dashed lines) and with  $\sqrt{s}$  (solid lines), in two in-medium versions: WRW (left panel) and including Pauli blocking and hadron self-energies (right panel).

The  $K^-$  absorption ( $\text{Im } V_{K^-}$ ) is suppressed in nuclear matter.

$\bar{K}$ -nuclear states probe energies 50-100 MeV below the  $\bar{K}N$  threshold!

## resume on $K^-$ in nuclear matter

chirally motivated  $K^- N$  amplitudes in a free space



in-medium  $K^- N$  amplitudes



$K^-$ -nuclear optical potential



solve the Klein-Gordon equation to get  $K^-$ -nuclear (or atomic) states

- accounting for **Pauli principle (and hadron self-energies)**
- **energy dependence** of the optical potential treated self-consistently
- **only processes on one nucleon accounted for** in the  $V_{\text{opt}}^{(1)} \approx t_{\bar{K}N} \rho$  potential

# Kaonic atoms analysis

*E. Friedman, A. Gal, NPA 959 (2017) 66 :*

- $\chi^2$  fits to kaonic atoms data (energy shifts, widths and yields = upper level widths) presented
- optical potentials  $V_{K^-} = V_{K^-}^{(1)}$  constructed from the chirally motivated amplitudes fail to describe the data
- $K^-$  interactions with two and more nucleons are needed, e.g.  $K^- + N + N \rightarrow Y + N$

$$2\text{Re}(\omega_{K^-})V_{K^-}^{(2)} = -4\pi B \left(\frac{\rho}{\rho_0}\right)^\alpha \rho$$

where  $B$  is a **complex** parameter and  $\alpha$  is positive

- total  $K^-$  optical potential  $V_{K^-} = V_{K^-}^{(1)} + V_{K^-}^{(2)}$

# Kaonic atoms analysis

Fits to 65 kaonic atoms data points when single-nucleon amplitudes are supplemented with a  $B(\rho/\rho_0)^\alpha \rho$  term representing multinucleon absorptions.

	P1	KM1	P2	KM2
$\alpha$	1	1	2	2
Re $B$ (fm)	$-1.3 \pm 0.2$	$-0.9 \pm 0.2$	$-0.5 \pm 0.6$	$0.3 \pm 0.7$
Im $B$ (fm)	$1.5 \pm 0.2$	$1.4 \pm 0.2$	$4.6 \pm 0.7$	$3.8 \pm 0.7$
$\chi^2(65)$	125	123	125	123
	B2	B4	M1	M2
$\alpha$	0.3	0.3	0.3	1
Re $B$ (fm)	$2.4 \pm 0.2$	$3.1 \pm 0.1$	$0.3 \pm 0.1$	$2.1 \pm 0.2$
Im $B$ (fm)	$0.8 \pm 0.1$	$0.8 \pm 0.1$	$0.8 \pm 0.1$	$1.2 \pm 0.2$
$\chi^2(65)$	111	105	121	109

All six  $\bar{K}N$  models acceptable?

# Kaonic atoms analysis

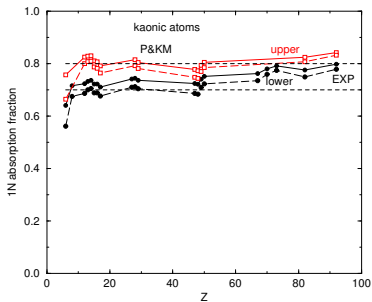
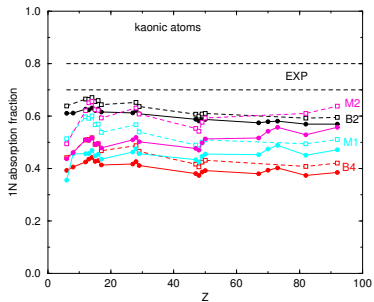
additional constrain introduced by Friedman and Gal,  
**single vs. multinucleon fraction** for absorption at rest

the experimental fractions of multinucleon absorptions at rest:

- $0.26 \pm 0.05$  (C, F, Br)  
*H. Davis et al., Nuovo Cimento 53 (1968) 313 (Berkeley)*
- $0.28 \pm 0.03$  (Ne)  
*J.W. Moulder et al., Nucl. Phys. B35 (1971) 332 (BNL)*
- $0.19 \pm 0.03$  (C)  
*C. Vander Velde-Wilquet et al., Nuovo Cimento 39A (1977) 538 (CERN)*

**Fraction of *single-nucleon* absorption  $0.75 \pm 0.05$  (average value) applied in the kaonic atoms analysis.**

# Kaonic atoms analysis



Fraction of single-nucleon absorption for the chiral approaches. Solid circles for lower states, open squares for upper states. Left: B2, B4, M1 and M2, Right: P1, KM1 for  $\alpha = 1$  (solid) and P2, KM2 for  $\alpha = 2$  (dashed).

Only P and KM models found acceptable by the F+G analysis.  
Preliminarily, the BCN model is acceptable too.

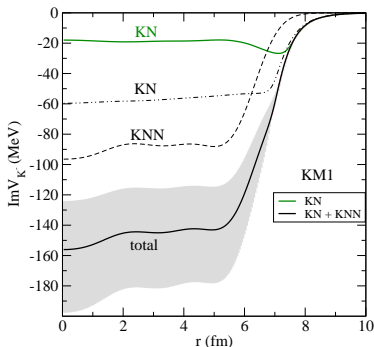
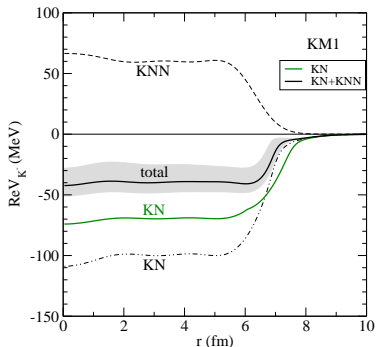
# $K^-$ -nuclear quasi-bound states

- In calculations of **kaonic nuclei**, the nucleus is described within an **RMF model**, *D. Gazda, J. Mareš, Nucl. Phys. A 881 (2012) 159*
- $ImB$  multiplied by a kinematical suppression factor to account for phase space reduction.
- The kaonic atoms analysis also demonstrated that the  $K^-$  optical potential is reasonably well determined only up to  $\rho \sim 0.5\rho_0$ .
- Two limiting cases for  $V_{K^-}^{(2)}$  considered in calculations of kaonic nuclei:
  - full density option (**FD**) –  $B(\rho/\rho_0)^\alpha \rho$  form in the entire nucleus
  - half density limit (**HD**) – fix  $V_{K^-}^{(2)}$  at constant value  $V_{K^-}^{(2)}(0.5\rho_0)$  for  $\rho(r) \geq 0.5\rho_0$

*J. Hrtánková, J. Mareš, PLB 770 (2017) 342; PRC 96, 015205 (2017)*

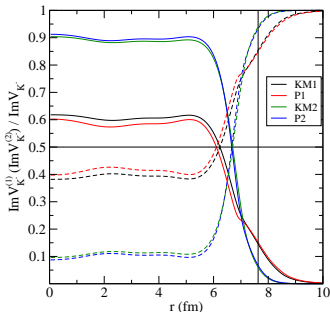
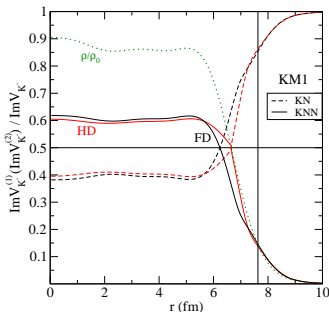


# Total $K^-$ optical potential



The respective contributions from  $K^-N$  and  $K^-NN$  potentials to the total real and imaginary  $K^-$  optical potential in the  $^{208}\text{Pb}+K^-$  nucleus, calculated self-consistently in the KM1 model and for the FD variant. The **single-nucleon  $K^-$  potential** (green solid line) of the KM model is shown for comparison. Shaded area = uncertainties in the  $KNN$  part input.

## $K^-N$ vs. $K^-NN$ absorption in $^{208}\text{Pb}$



Ratios of  $\text{Im}V_{K^-}^{(1)}$  and  $\text{Im}V_{K^-}^{(2)}$  potentials to the total  $\text{Im}V_{K^-}$  as a function of radius, calculated self-consistently for the  $^{208}\text{Pb}+K^-$  system. The vertical lines mark the nuclear surface density of  $0.15 \rho_0$ .

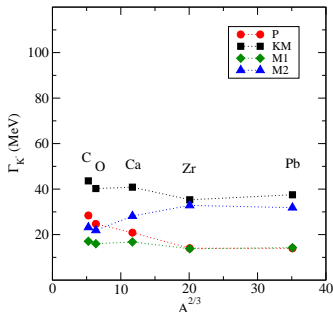
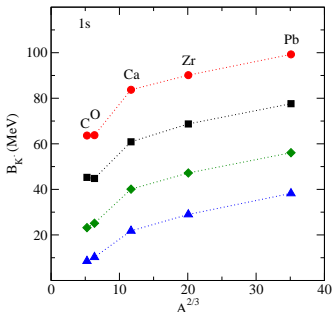
Left: HD and FD versions compared for the KM1 model.

Right: the KM and P models compared in the FD version.

$\bar{K}NN$  absorption dominates in nuclear interior, the  $\bar{K}N$  absorption at low densities.

# $K^-$ 1s binding energies and widths

First step:  $V_{\text{opt}} = V_{K^-}^{(1)}$ , the  $\bar{K}N$  interactions provided by the P, KM, M1, and M2 models, self-consistency ( $\sqrt{s}$  energy, binding energy  $B_{K^-}$ ) applied



Result: Relatively deeply bound states with *reasonable widths*, **but** ...

# $K^-$ 1s binding energies and widths

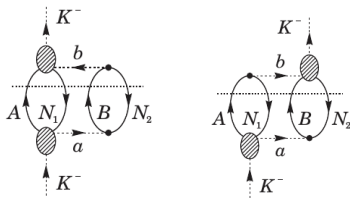
... when the  $K^-NN$  phenomenological term  $V_{K^-}^{(2)} \sim B(\rho/\rho_0)^\alpha$  ( $\alpha = 1$  or  $2$ ) is added, some states are not bound and the absorption widths become larger than the pertinent binding energies  $\Rightarrow$   $K^-$ -nuclear states are not likely (at least not for  $A \geq 12$ )

KM model		$\alpha = 1$		$\alpha = 2$		
		KN	HD	FD	HD	FD
$^{16}\text{O}$	$B_{K^-}$	45	34	not	48	not
	$\Gamma_{K^-}$	40	109	bound	121	bound
$^{40}\text{Ca}$	$B_{K^-}$	59	50	not	64	not
	$\Gamma_{K^-}$	37	113	bound	126	bound
$^{208}\text{Pb}$	$B_{K^-}$	78	64	33	80	53
	$\Gamma_{K^-}$	38	108	273	122	429
P model		$\alpha = 1$		$\alpha = 2$		
$^{16}\text{O}$	$B_{K^-}$	64	49	not	63	not
	$\Gamma_{K^-}$	25	94	bound	117	bound
$^{40}\text{Ca}$	$B_{K^-}$	81	67	not	82	not
	$\Gamma_{K^-}$	14	95	bound	120	bound
$^{208}\text{Pb}$	$B_{K^-}$	99	82	36	96	47
	$\Gamma_{K^-}$	14	92	302	117	412

# Microscopic model for in-medium $K^- NN$ absorption

Can we improve on the  $K^- NN$  absorption? Earlier microscopic treatment in  
*T. Sekihara et al. - PRC 86, 065205 (2012)* for the  $K^- NN$ , free space  $\bar{K}N$  amplitudes  
*H. Nagahiro et al. - PLB 709, 87 (2012)* for the  $\eta' NN$  system

New work on the  $\bar{K} NN$  system employs the **in-medium BCN and P amplitudes**:  
*J. Hrtánková, À. Ramos - prepared for a publication (2019)*



- $K^- NN$  self-energy

$$\begin{aligned} \Pi_{AB}(\vec{q}, q_0) &= -it_{K^- N_1 \rightarrow Aa} t_{K^- N_1 \rightarrow Ab}^* V_{aBN_2} V_{bBN_2} \cdot \\ &\cdot \int \frac{d^4 q}{(2\pi)^4} U_{AN_1}(p-q) U_{BN_2}(q) q^2 \frac{1}{q^2 - m_a^2} \frac{1}{q^2 - m_b^2} \end{aligned}$$

- t-matrices taken from chiral meson-baryon models

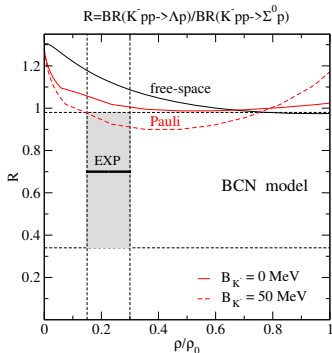
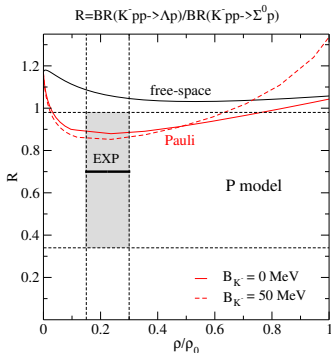
# $K^- NN$ absorption - sample (preliminary!) results

- AMADEUS measurement of the  $\Lambda p$  to  $\Sigma^0 p$  production rate in  $K^- NN$  QF absorption

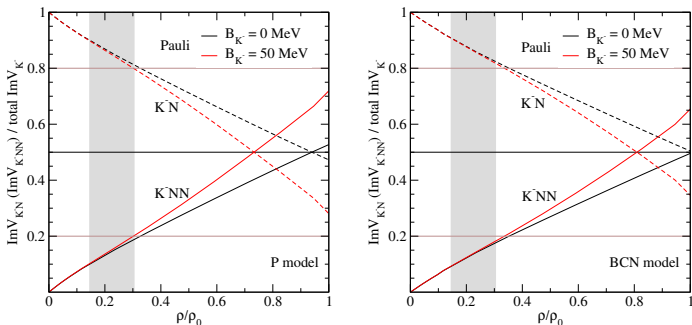
$$\mathcal{R} = \frac{\text{BR}(K^- pp \rightarrow \Lambda p)}{\text{BR}(K^- pp \rightarrow \Sigma^0 p)} = 0.7 \pm 0.2(\text{stat.})_{-0.3}^{+0.2}(\text{syst.})$$

*R. Del Grande et al. - Eur. Phys. J. C79 (2019) 190*

- two settings of the  $K^- NN$  model used assuming  $B_{K^-} = 0$  and 50 MeV; the Pauli blocked (WRW) amplitudes reproduce the experimental rate, the free space amplitudes do not  $\Rightarrow$  **medium effects are relevant!**



# $K^-NN$ absorption - sample (preliminary!) results



Ratio of single nucleon ( $K^-N$ ) and two-nucleon ( $K^-NN$ ) absorptive potentials to the total absorptive potential ( $\text{Im}V_{K^-}$ ). The grey bouds shows the region of densities probed by low energy  $K^-$ . Left - P model, right - BCN model.

$B_K = 0$  MeV -  $\bar{K}NN$  starts to dominate at  $\rho \approx \rho_0$

$B_K = 50$  MeV -  $\bar{K}NN$  starts to dominate at  $\rho \approx 0.7 - 0.8 \rho_0$

# Summary

- The up-to-date (NLO) chirally motivated  $\bar{K}N$  models provide very different predictions for the  $K^-N$  amplitudes at subthreshold energies.
- In-medium kaons probe energies as far as 50-100 MeV below the  $\bar{K}N$  threshold. A realistic treatment of the energy dependence including the Pauli blocking and hadron self-energies is essential.
- Fits to kaonic atoms demonstrate the need of NN (or multinucleon) contribution to the  $K^-$ -nuclear optical potential. The P and KM models are favoured satisfying the additional constrain of 1N to 2N absorption rate. The new BCM model is likely to do well in this respect too.
- The inclusion of multinucleon absorption in the RMF calculations of  $K^-$  quasi-bound states in many-body systems leads to huge widths, considerably exceeding the binding energies. If this feature is confirmed the observation of such states is unlikely. The conclusion does not apply to few body  $K^-$ -nucleons systems.
- Microscopical model for  $K^-NN$  absorption in nuclear matter has been developed using chiral  $\bar{K}N$  amplitudes. The preliminary results look encouraging, the ratio of  $\Lambda p$  to  $\Sigma^0 p$  production measured by AMADEUS is reproduced by the model when the in-medium amplitudes are employed.