Spectroscopy from the lattice:
the scalar glueball and $K\pi$ scattering

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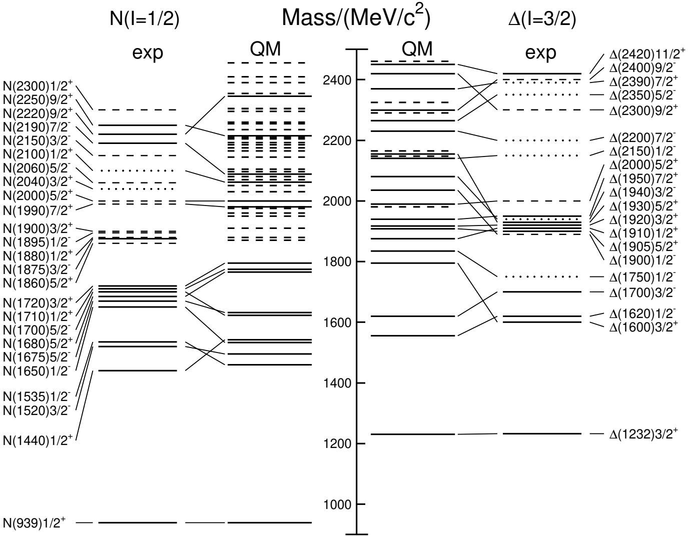
June 6, 2019

MENU 2019

Motivation

- significant experimental efforts to map out QCD spectrum: Hall B/D (JLab), COMPASS (CERN), etc.
- few states (QCD)-stable: $\pi, K, N, \Lambda, \Sigma, \Xi, \Omega$
  - most are unstable resonances
- theoretical description of states important, many poorly understood
  - eg. tetraquarks ($qq\overline{q}\overline{q}$), glueballs (bound gluons), $\ldots$
- study low-lying hadronic spectrum using lattice QCD:
  - QCD stationary states in finite-vol.
  - Lüscher: $2 \to 2$ scattering amplitudes from finite-vol. energies

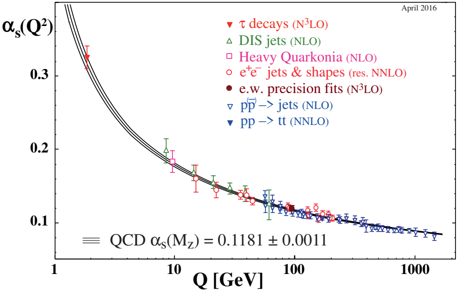
Experiment vs. Quark Model



Particle Data Group

QCD on the Lattice

- strong coupling, α_s, large at low energies
  - $\rightarrow$ non-perturbative



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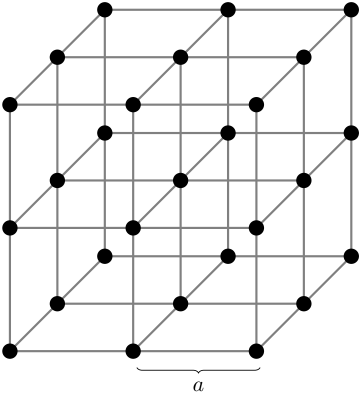
- Lattice QCD : define theory on finite, Euclidean, space-time lattice
  - lattice spacing $\rightarrow$ regulator

- observables:

$$\langle A \rangle = \frac{1}{Z} \int \mathcal{D}[\overline{\psi}, \psi, U] A \, e^{-S[\overline{\psi}, \psi, U]}$$

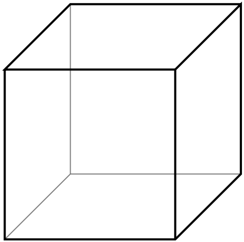


- evaluate numerically via Monte Carlo

  $\Rightarrow$ control over (many) systematic errors, etc.

Rotational Symmetry

- periodic B.C. in cubic box:
  - $\Rightarrow$ J^{PC} no longer good quantum numbers



- label stationary states using irreps of cubic symmetry group O_h
  - parity (u/g) and G-parity ($+/-$) where relevant

- continuum spin ID:

J	A_1	A_2	E	T_1	T_2	J	G_1	G_2	H
0	1	0	0	0	0	$\frac{1}{2}$	1	0	0
1	0	0	0	1	0	$\frac{3}{2}$	0	0	1
2	0	0	1	0	1	$\frac{5}{2}$	0	1	1
3	0	1	0	1	1	$\frac{7}{2}$	1	1	1
4	1	0	1	1	1	$\frac{9}{2}$	1	0	2

Extracting the Finite-Volume Spectrum

- temporal. correlation matrix:

$$C_{\alpha\beta}(t) \equiv \langle 0|\mathcal{O}_\alpha(t+t_0)\overline{\mathcal{O}}_\beta(t_0)|0\rangle$$
$$= \sum_n \langle 0|\mathcal{O}_\alpha|n\rangle\langle n|\overline{\mathcal{O}}_\beta|0\rangle\, e^{-E_n t}$$

- eigenvalues tend to lowest N energies

$$\lim_{t\to\infty} \lambda_n(t) = b_n e^{-E_n t}[1+\mathcal{O}(e^{\Delta_n t})]$$
$$\Rightarrow E^n_{\text{eff}}(t) = \frac{1}{\Delta t}\ln\left(\frac{\lambda_n(t)}{\lambda_n(t+\Delta t)}\right)$$

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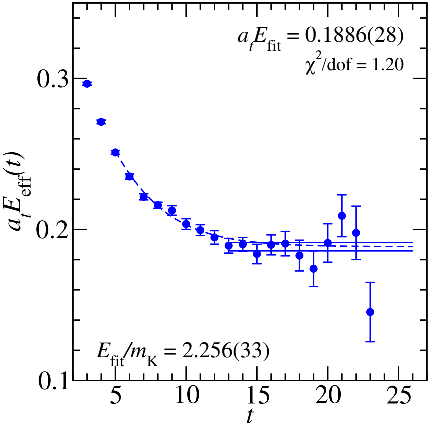

quality of signal/excited state contamination *highly* dependant on operator basis

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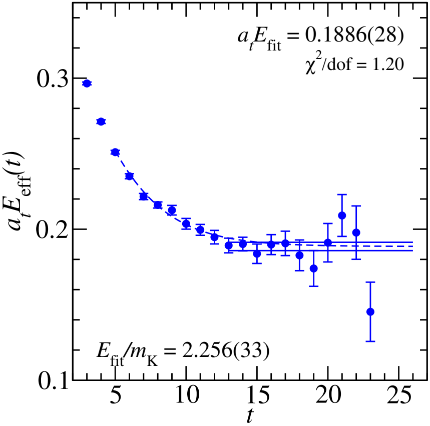


quality of signal/excited state contamination *highly* dependant on operator basis

careful operator design is crucial

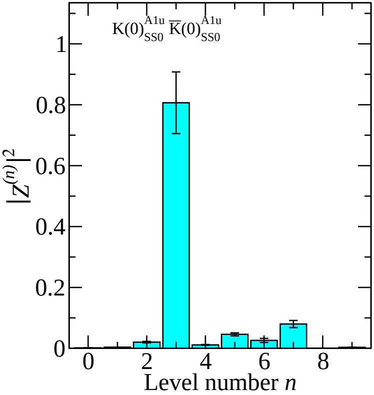
Extracting the Finite-Volume Spectrum

- 2-pt. correlation matrix:

$$C_{\alpha\beta}(t) = \sum_n \langle 0|\mathcal{O}_\alpha|n\rangle\langle n|\overline{\mathcal{O}}_\beta|0\rangle\, e^{-E_n t}$$
$$= \sum_n Z_\alpha^{(n)} Z_\beta^{(n)*}\, e^{-E_n t}$$

- level ID inferred from Z overlaps with *probe* operators:

$$|\Phi_j\rangle \equiv \mathcal{O}_j|0\rangle \quad \Rightarrow \quad Z_j^{(n)} = \langle \Phi_j|n\rangle$$



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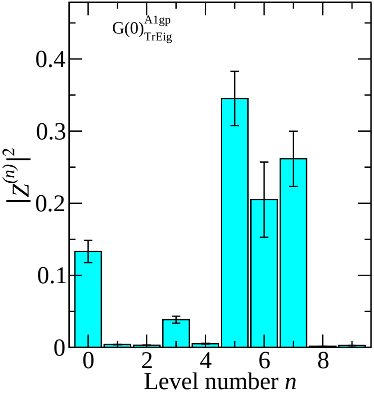

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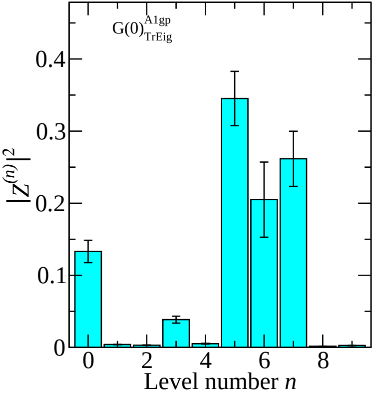
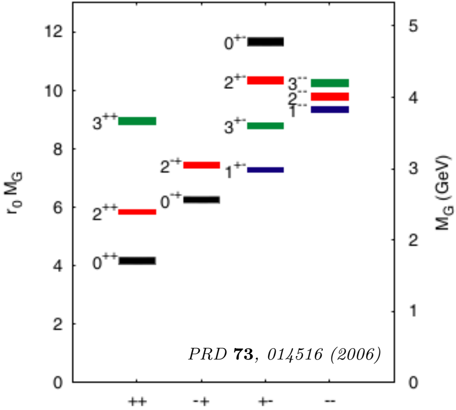


multiple operators can mix/overlap on any given eigenstate

overlaps give **qualitative** measure of mixing between states

The Scalar Glueball

- glueball: hypothetical bound state of gluons
- experimental evidence elusive, light scalar candidates:
  - $f_0(1370), f_0(1500), f_0(1710)$
- lattice studies to date:
  - light scalar ~ 1700 MeV
  - most in pure $SU(3)$/quenched approx. (no quark/meson mixing)



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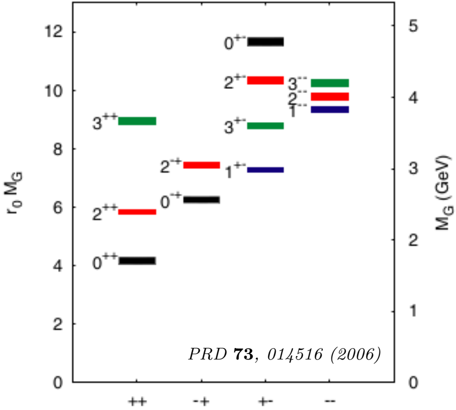
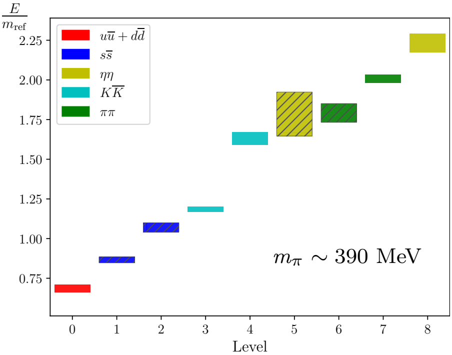


PRD **73**, *014516 (2006)*

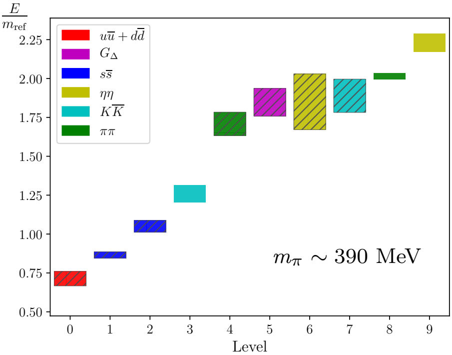
- here: extract low-lying A_{1g}^{+} spectrum with $q\bar{q}$, meson-meson, & glueball operators
    - first look (from the lattice) at mixing between glueball, $q\bar{q}$, and two-hadron states

A_{1g}^{+} Spectrum

$24^3 \times 128$ anisotropic lattice, $m_\pi \sim 390$ MeV, $m_K \sim 550$ MeV:



w/o glueball operator

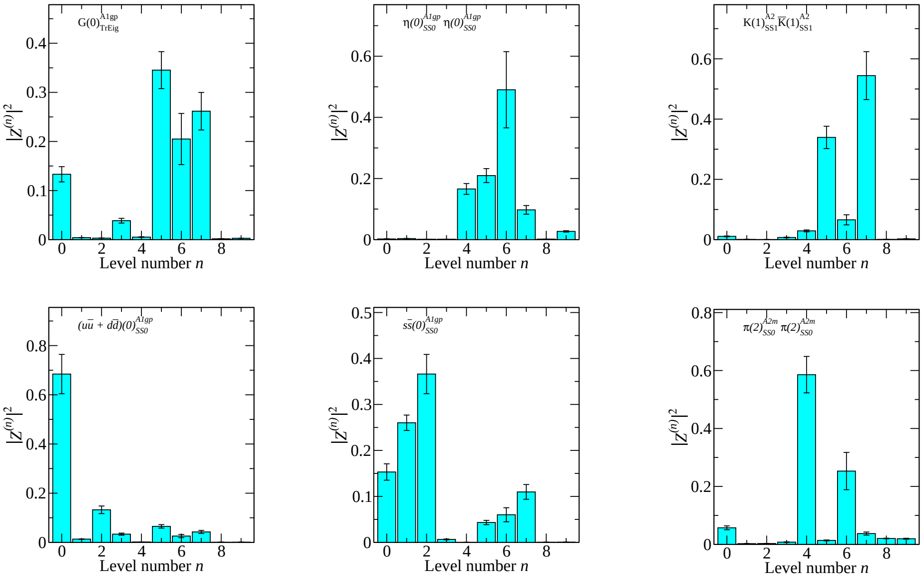


with glueball operator

hatched boxes: significant overlap with multiple operators

$m_{\text{ref}} = 1.82 m_K \sim 1$ GeV

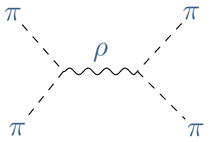
A_{1g}^{+} Overlaps

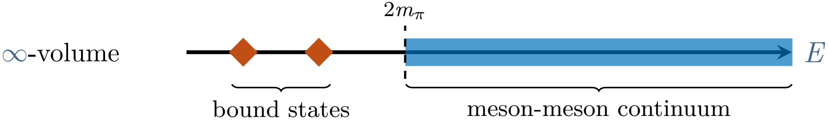


Infinite-Volume Physics

Scattering process: eg.

$I = 1 \quad \pi\pi \to \pi\pi$





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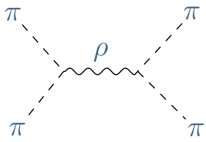

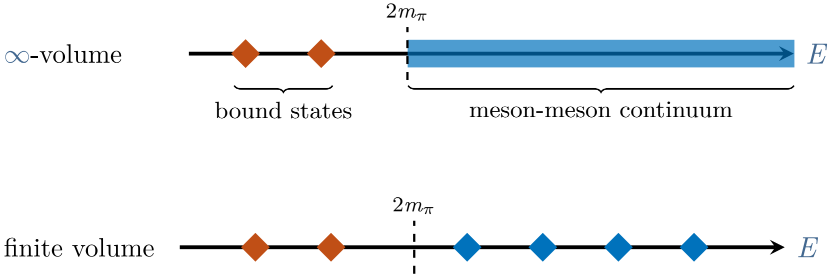

no continuum of scattering states → how to access ∞-vol. physics?

$$\boldsymbol{p} = \frac{2\pi}{L}\boldsymbol{d}$$

Lüscher Quantisation ($2 \to 2$)

Infinite volume physics from LQCD

Lüscher: Relationship between finite volume spectrum and infinite volume scattering matrix

- Quantisation condition:

$$\det[\, \widetilde{K}^{-1} - B \,] = 0$$

- For each E_{cm} in spectrum, determinant gives single relation to entire scattering matrix
  - ⇒ Exactly solvable for single channel, single partial wave
  - ⇒ ℓ mixing/coupled decay channels requires parameterisation of $\widetilde{K}$ and a fit

[Nucl. Phys. B **924** 447-507 (2017)]

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box matrix: known function of $(E_{\rm cm}, L)$

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Elastic $K\pi$ Scattering

- $32^3 \times 256$ anisotropic lattice, $m_\pi \approx 230$ MeV
- include $\ell = 0, 1, 2$ partial waves
- fit forms

$$
\begin{aligned}
(\widetilde{K}^{-1})^{\mathrm{ERE}}_{00} &= \frac{-1}{m_\pi a_0} + \frac{m_\pi r_0}{2}\frac{\boldsymbol{q}^2_{\mathrm{cm}}}{m_\pi^2} \\
(\widetilde{K}^{-1})^{\mathrm{BW}}_{11} &= \frac{6\pi E_{\mathrm{cm}}}{g^2 m_\pi}\left(\frac{m^2_{K^*}}{m_\pi^2} - \frac{E^2_{\mathrm{cm}}}{m_\pi^2}\right) \\
(\widetilde{K}^{-1})^{\mathrm{ERE}}_{22} &= \frac{-1}{m_\pi^5 a_2}
\end{aligned}
$$

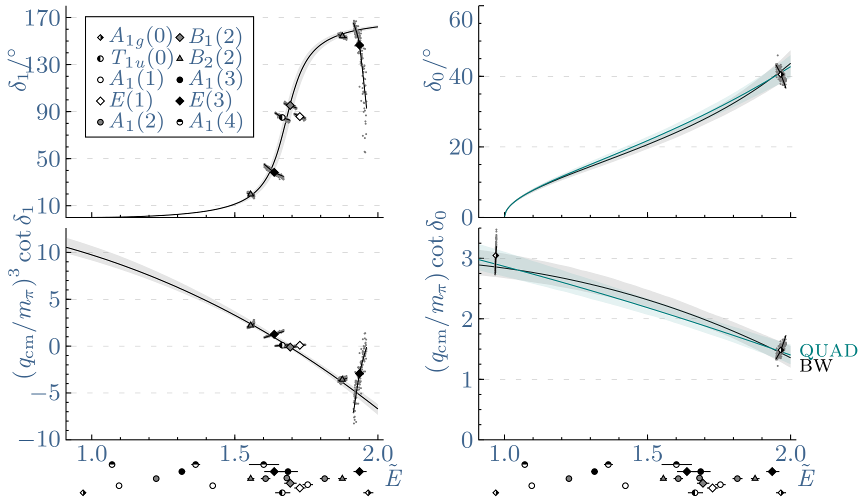
- p-wave : $K^*(892)$
- s-wave : $K_0^*(700)/\kappa$

$\mathbf{d}$	Λ	ℓ
$(0,0,0)$	A_{1g}	$0, 4, \dots$
	T_{1u}	$1, 3, \dots$
$(0,0,n)$	A_1	$0, 1, 2, \dots$
	E	$1, 2, 3, \dots$
$(0,n,n)$	A_1	$0, 1, 2, \dots$
	B_1	$1, 2, 3, \dots$
	B_2	$1, 2, 3, \dots$
(n,n,n)	A_1	$0, 1, 2, \dots$
	E	$1, 2, 3, \dots$

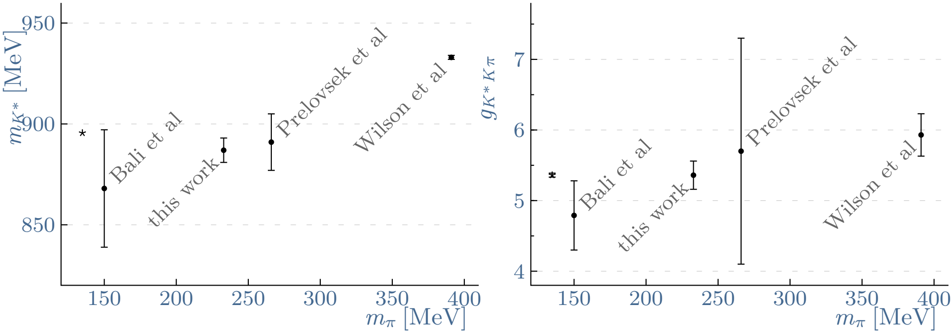
[Nucl.Phys. B **932** 29-51 (2018)]

$K\pi$ Scattering Phase-Shifts

- plots of p-wave and s-wave phase shift ($\tilde{E} = (E_{\rm cm} - m_K)/m_\pi$)
- d-wave contributes negligibly



- comparison with (some) lattice calculations of $K^*(892)$ resonance parameters
- phenomenological values shown with asterisks



Conclusions/Future work - Scattering

- goal: calculate scattering phase-shifts/resonance parameters from LQCD
- meson-meson scattering at a mature stage
  - see B. Hörz's talk today
  - physical point calculations feasible - large volumes required
- meson-baryon scattering now possible: eg. $N\pi$: PRD **97**, 014506 (2018)
  - see J. Bulava's plenary this morning
  - baryon-baryon at light quark masses on the horizon
- extension of formalism to include three-body channels underway
  - see M. Mai's talk on Mon.

Conclusions/Future work - Spectroscopy

- goal: *qualitative* description of resonance spectrum

- high computational cost for large operator bases/volumes
  - careful operator design crucial

- $\overline{q}q$ states straightforward but many interesting states not well described
  - ⇒ hybrids
  - ⇒ molecular states
  - ⇒ ...

- influence of tetraquark operators: κ and $a_0(980)$
  - see D. Darvish's talk on Tues.

Meson and Baryon elemental operators

- q's = smeared, displaced quark fields

$$\overline{\Phi}^{AB}_{\alpha\beta;ij}(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}+\frac{1}{2}(\boldsymbol{d}_\alpha+\boldsymbol{d}_\beta))}\delta_{ab}\,\overline{q}^{B}_{b\beta j}(\boldsymbol{x},t)\,q^{A}_{a\alpha i}(\boldsymbol{x},t)$$

$$\overline{\Phi}^{ABC}_{\alpha\beta\gamma;ijk}(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\varepsilon_{abc}\,\overline{q}^{C}_{c\gamma k}(\boldsymbol{x},t)\,\overline{q}^{B}_{b\beta j}(\boldsymbol{x},t)\,\overline{q}^{A}_{a\alpha i}(\boldsymbol{x},t)$$

- group-theory projections onto irreps of lattice symmetry group

$$\overline{M}_l(t) = c^{(l)*}_{\alpha\beta}\overline{\Phi}^{AB}_{\alpha\beta}(t) \qquad \overline{B}_l(t) = c^{(l)*}_{\alpha\beta\gamma}\overline{\Phi}^{ABC}_{\alpha\beta\gamma}(t)$$

More exotic operators

- tetraquarks: $3 \times 3 \times \overline{3} \times \overline{3} = 1 + 1 + 8 + 8 + \ldots$

$$\overline{\Phi}^{ABCD\pm}_{\alpha\beta\mu\nu;ijkl}(\boldsymbol{p}, t) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} (\delta_{ab}\delta_{cd} \pm \delta_{ad}\delta_{bc}) \times \overline{q}^{D}_{d\nu l}(\boldsymbol{x}, t)\, q^{C}_{c\mu k}(\boldsymbol{x}, t)\, \overline{q}^{B}_{b\beta j}(\boldsymbol{x}, t)\, q^{A}_{a\alpha i}(\boldsymbol{x}, t)$$

→ preprint coming soon: κ & $a_0(980)$ resonances

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two colour singlets!

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- scalar glueball: (purely gluonic - no quark fields)

$$G_{\Delta} \sim \mathrm{Tr}\,\tilde{\Delta}, \qquad \tilde{\Delta} \equiv \text{covariant laplacian}$$

$\rightarrow$ see PRD **88**, 014511 (2013)

S-wave $K\pi$ amplitude: $K_0^*(700)/\kappa$

$$\frac{m_{K^*}}{m_\pi} = 3.808(18), \quad g = 5.33(20), \quad m_\pi a_0 = -0.353(25),$$
$$m_\pi^5 a_2 = -0.0013(68), \qquad \chi^2/\text{dof} = 1.42$$

- based on LO ERE, $m_\pi a_0 < 0$ suggests virtual bound state
- however, NLO parameters give $1 - 2r_0/a_0 = -8.9(2.4)$ which must be > 0 for a (real or virtual) bound state
- zeros of $\boldsymbol{q}_{\text{cm}} \cot \delta_0 - i\boldsymbol{q}_{\text{cm}}$: $m_R/m_\pi = 4.66(13) - 0.87(18)i$
  - consistent with BW fit
- better energy resolution & careful analytic continuation required

[Nucl.Phys. B932 (2018)]