Proton scalar polarizabilities from real Compton scattering data

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WHAT?
Extraction of dipole scalar polarizabilities (electric and magnetic) from proton RCS data

WHEN & WHERE?
During my PhD in Physics at Università degli Studi di Pavia (Italy) & INFN

HOW?
Dispersion Relation approach + data analysis (bootstrap)

WHY?
“...sure, it may give some practical results, but that's not why we do it” - R. P. Feynman
1. PROTON with DISPERSION RELATIONS
Real Compton scattering (RCS)

Proton target + REAL photon in & REAL photon out

\[ \gamma(q) + N(p) \rightarrow \gamma(q') + N(p') \]
Real Compton scattering (RCS)

Proton target + REAL photon in & REAL photon out

\[ \gamma(q) + N(p) \rightarrow \gamma(q') + N(p') \]

Check the response of the proton to the external quasi-static field (the photon)

See Measurement of the proton polarizabilities at MAMI – E. Mornacchi
Real Compton scattering (RCS)

Proton target + \textbf{REAL} photon in & \textbf{REAL} photon out

\[ \gamma(q) + N(p) \rightarrow \gamma(q') + N(p') \]

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The proton structure in RCS

Powell cross section: point-like nucleon with anomalous magnetic moment

Static polarizabilities: response of the internal nucleon degrees of freedom to a static electric and magnetic field

\[ \gamma p \rightarrow \gamma p \ (\text{MAMI}) \]

\[ \theta_{\text{lab}} = 59^\circ \]

The proton structure in RCS

**Powell** cross section: point-like nucleon with anomalous magnetic moment

**Static polarizabilities**: response of the internal nucleon degrees of freedom to a *static* electric and magnetic field

\[
\begin{align*}
\frac{d\sigma}{d\Omega}_{\text{cm}} & = 2\pi \left[ \omega^2 \left( a_{E1} \vec{E}^2 + b_{M1} \vec{B}^2 \right) \right. \\
& \quad + \omega^3 \left( \gamma_{E1E1} \vec{\sigma} \cdot (\vec{E} \times \vec{\dot{E}}) + \gamma_{M1M1} \vec{\sigma} \cdot (\vec{B} \times \vec{\dot{B}}) \right) \\
& \quad - 2 \gamma_{M1E2} \sigma_i B_j E_{ij} + 2 \gamma_{E1M2} \sigma_i E_j B_{ij} \left] + \mathcal{O}(\omega^3) \right) 
\end{align*}
\]

Unitarity & analyticity: the basis of DRs

Unitarity: the sum over all possible processes has probability = 1

\[ S = I + i T \]

\[ 2 \Im T_{fi} = \sum_n T^*_f n T_{ni} \]

Sum over all the intermediate states!
Unitarity & analyticity: the basis of DRs

Unitarity: the sum over all possible processes has probability $= 1$

\[ S = I + i T \]

\[ 2 \sum T_{fi} = \sum_{n} T_{fn}^* T_{ni} \]

Sum over all the intermediate states!

\[ f(z) = \frac{1}{2\pi i} \oint_{C} \frac{f(s)}{s-z} \, ds \]
Unitarity: the sum over all possible processes has probability = 1

\[ S = I + iT \]

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Sum over all the intermediate states!

Unitarity & analyticity: the basis of DRs

\[ f(z) = \frac{1}{2\pi i} \oint_C \frac{f(s)}{s-z} ds \]

Branch cuts due to inelasticity
Inspecting the proton

RCS differential cross section $\rightarrow 6$ amplitudes $A_i$

$v \rightarrow$ energy $\quad t \rightarrow$ transferred momentum

$$A_i(v,t) = A_i^B(v,t) + \int_{v_{thr}}^{v_{MAX}} \ldots + \int \ldots$$

Inspecting the proton

RCS differential cross section → 6 amplitudes $A_i$

$\nu \rightarrow$ energy $\qquad t \rightarrow$ transferred momentum

$$A_i(\nu, t) = A^B_i(\nu, t) + \int_{\nu_{thr}}^{\nu_{MAX}} \ldots + \int \ldots$$

For $i = 3, \ldots, 6$: “good” behavior

$$A_i(\nu, t) = A^B_i(\nu, t) + \int_{\nu_{thr}}^{\infty} \ldots + 0$$

For $i = 1, 2$: “bad” behavior

$$A_i(\nu, t) = A^B_i(\nu, t) + \int_{\nu_{thr}}^{\nu_{MAX}} \ldots + A^AS_i$$

Inspecting the proton

RCS differential cross section $\rightarrow$ 6 amplitudes $A_i$

$v \rightarrow$ energy $t \rightarrow$ transferred momentum

$$A_i(v,t) = A_i^B(v,t) + \int_{\nu_{\text{thr}}}^{\nu_{\text{MAX}}} \ldots + \int_{\nu_{\text{thr}}}^{\nu_{\text{MAX}}} \ldots + 0$$

For $i=3,\ldots,6$: “good” behavior

$$A_i(v,t) = A_i^B(v,t) + \int_{\nu_{\text{thr}}}^{\nu_{\text{MAX}}} \ldots + A_i^{\text{AS}}$$

For $i=1,2$: “bad” behavior

Asymptotic contribution $\rightarrow$ meson exchange

Contour behavior: that's the problem! $\rightarrow$ faster convergence is needed...

SUBTRACTED DISPERSION RELATIONS

$A_i(0,0) = a_i$ Static polarizabilities

$A_i(\nu, t) = A^s_i(\nu, 0) + A^t_i(0, t) + A_i(0, 0)$
A_i(0,0) = a_i

Static polarizabilities

\[ A_i (\nu, t) = A_i^s (\nu, 0) + A_i^t (0, t) + A_i (0, 0) \]

Subtracted Dispersion Relations (s-channel)

\[ A_i^s (\nu, 0) = \frac{2}{\pi} \nu^2 \text{P} \int_{\nu_{thr}}^{\infty} \text{Im}_s A_i (\nu', t) \frac{d\nu'}{\nu' (\nu'^2 - \nu^2)} \]

Dispersion Relations & polarizabilities

\[ A_i(\nu, t) = A^s_i(\nu, 0) + A^t_i(0, t) + A_i(0, 0) \]

Subtracted Dispersion Relations (s-channel)

\[ A^s_i(\nu, 0) = \frac{2}{\pi} \frac{\nu^2}{\nu'^2 - \nu^2} P \int_{\nu_{th}}^{\infty} \text{Im} \, A_i(\nu', t) \, \frac{d\nu'}{\nu' (\nu'^2 - \nu^2)} \]

s-CHANNEL

\[ \text{Im} = \pi \pi N + \ldots \]

A naive picture of the polarizabilities

\[ \alpha_{E1} \]

\[ \text{ELECTRIC FIELD} \]
A naive picture of the polarizabilities

$\alpha_{E1}$
A naive picture of the polarizabilities

The proton is 1000 times “electrically” stiffer than the Hydrogen!
A naive picture of the polarizabilities

\[ \alpha_{E1} \quad \beta_{M1} \]

ELECTRIC FIELD     Electric dipole moment      MAGNETIC FIELD

The proton is **1000 times** “electrically” stiffer than the Hydrogen!
The proton is 1000 times “electrically” stiffer than the Hydrogen!
A naive picture of the polarizabilities

\[ \alpha_{E1} \]

\[ \beta_{M1} \]

The proton is **1000 times** “electrically” stiffer than the Hydrogen!
2. TRADITIONAL FITS
How fits are usually done

Experimental points assumed Gaussian distributed around the model predictions (in the best value of the parameters)

\[ E_i \in G\left[ \hat{T}_i, \sigma_i^2 \right] \]

\[ \chi^2 = \sum_i \left( \frac{E_i - \hat{T}_i}{\sigma_i} \right)^2 \]
How fits are usually done

Experimental points assumed Gaussian distributed around the model predictions (in the best value of the parameters)

$$\chi^2 = \sum_i \left( \frac{E_i - \hat{T}_i}{\sigma_i} \right)^2$$

Inclusion of the systematic sources of uncertainties

$$\chi^2 = \sum_{i,k} \left( \frac{f_k E_i - \hat{T}_i}{f_k \sigma_i} \right)^2 + \left( \frac{1 - f_k}{\Delta_k} \right)^2$$
Some pathological problems

Which is the probability distribution of the fitted parameters? A Gaussian?

How can we include the propagation of parameters that are not fitted? Squared sum?

Which probability distribution shall we use for the modified chi squared? A “traditional” one?
Some pathological problems

Which is the probability distribution of the fitted parameters? A Gaussian?

How can we include the propagation of parameters that are not fitted? Squared sum?

Which probability distribution shall we use for the \textit{modified chi squared}? A “traditional” one?

\textbf{SOLUTION}: change perspective and let the data define the rules!

3. OUR CHOICE: bootstrap
Bootstrap in a nutshell

Whatever is the probability distribution of the experimental point, sample from there!

\[
B_{ij} \in P \left[ E_i, \sigma_i^2, \Delta_k \right] \quad \chi_j^2 = \sum_i \left( \frac{B_{ij} - \hat{T}_{ij}}{\sigma_{ij}} \right)^2
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Bootstrap in a nutshell

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**DO** \ j = 1, N

Minimize the function and find the best value of the parameters.
Store it.

**ENDDO**
Whatever is the probability distribution of the experimental point, sample from there!

\[ \chi_j^2 = \sum_{i} \left( \frac{B_{ij} - \hat{T}_{ij}}{\sigma_{ij}} \right)^2 \]

\[ B_{ij} \in P \left[ E_i, \sigma_i^2, \Delta_k \right] \]

\[ \text{DO j = 1, N} \]

Minimize the function and find the best value of the parameters.
Store it.

\[ \text{ENDDO} \]

We can reconstruct the **probability distribution** of the fitted parameters!
Our bootstrap sampling

Bootstrapped points assumed Gaussian distributed around the measured value

\[ B_{ij} = E_i + \gamma_{ij} \sigma_i \]

\[ B_{ij} \in G \left[ E_i, \sigma_i^2 \right] \]
Our bootstrap sampling

Bootstrapped points assumed Gaussian distributed around the measured value

\[ B_{ij} = E_i + \gamma_{ij} \sigma_i \]

\[ B_{ij} \in G\left[ E_i, \sigma_i^2 \right] \]

Additional shift due to systematic effect

\[ B_{ij} = \left( 1 + \delta_{ij} \right) \left( E_i + \gamma_{ij} \sigma_i \right) \]

\[ B_{ij} \in G\left[ E_i, \sigma_i^2 \right] \otimes U\left[ -\Delta_k, \Delta_k \right] \]
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The only assumption: which probability distribution for the systematic errors?
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The only assumption: which probability distribution for the systematic errors?

The sampling when more than one systematic source of error is included can be obtained from a convolution of all the sources.
Bootstrap: pros & contra

- Straightforward inclusion of systematic errors
- No assumptions on the parameters probability distributions
- Easy propagation of errors (for non-fitted parameters)
- Reconstruction of a realistic limit probability distribution for the \textit{chi squared} (see paper)

P. Pedroni, S. Sconfietti, \textit{in preparation}
Bootstrap: pros & contra

- Straightforward inclusion of systematic errors
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- Reconstruction of a realistic limit probability distribution for the *chi squared* (see paper)

- The number of iteration has to be big (from here on, N=10000 for me)

P. Pedroni, S. Sconfietti, in preparation
4. RESULTS & DISCUSSION
Our fitting framework

Fitting parameter: $a_{E1} - \beta_{M1}$

$a_{E1} + \beta_{M1}$ constrained
(Baldin’s sum rule)

Spin polarizabilities ($\psi_s$) fixed from experimental values (with errors)

Low-energy (below 150 MeV) RCS data

Our fitting framework

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Spin polarizabilities ($\gamma_s$) fixed from experimental values (with errors)

Low-energy (below 150 MeV) RCS data

There is some discussion on the “definition” of the Compton data set → let’s discuss at coffee breaks if you want!

Probability distributions

Non-Gaussian shape (not that far from that, indeed)

Systematic errors → enlarging the shape (here, not always)

The $\chi^2$ cumulative distribution

P. Pedroni, S. Sconfietti, *in preparation*
B. Pasquini, P. Pedroni, S. Sconfietti, arXiv:1903.07952,
to be submitted to J. Phys. G
Differential cross section (with errors)

Error band at 68% CL, from the parameters probability distribution!

Our results – overview

![Graph showing results comparison]

- Ziegler et al.
- Baldin SR
- PDG
- Olmos de Leon et al.
- Federspiel et al.
- Mc Govern et al.
- Mc Gibbon et al.
- Lensky et al.
- Our work

Take-home messages

1. Extraction of electric and magnetic polarizabilities of the proton from RCS data, with DRs and a detailed error propagation
Take-home messages

1. Extraction of electric and magnetic polarizabilities of the proton from RCS data, with DRs and a detailed error propagation

2. Use bootstrap for your fits!
\[
A(s) = \frac{1}{2\pi i} \oint_C \frac{A(z)}{z-s} \, dz \\
A(0) = \frac{1}{2\pi i} \oint_C \frac{A(z)}{z} \, dz \\
A(s) - A(0) = \frac{1}{2\pi i} \left[ \oint_C \frac{A(z)}{z-s} \, dz - \oint_C \frac{A(z)}{z} \, dz \right] \\
A(s) = A(0) + \frac{s}{2\pi i} \oint_C \frac{A(z)}{z(z-s)} \, dz
\]
\[ \frac{d\sigma}{d\Omega} \text{ (nb/sr)} \]

\[ E_\gamma \text{ (MeV)} \]

\[ \theta_{\text{lab}} = 45^\circ \]