A percent-level determination of the nucleon axial coupling from Quantum Chromodynamics

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Outline

• Lattice QCD Details
• Feynman-Hellman method
• Analysis and Results
• Summary
Nucleon Axial Coupling

Neutron life time:

Dubbers, Saul, Märkisch, Soldner, and Abele 2019

\[
\tau_{\text{neutron}} = \frac{5172.3(1.1)}{1 + 3g_A^2}
\]

If neutron decays to dark matter \( t_{\text{bottle}} < t_{\text{beam}} \)

Benchmark quantity for LQCD:
Strength of the attractive long-range force between nucleons determine by \( g_A \)
Controlling systematic errors have proved to be challenging.

One of the easiest nucleon quantities to compute, should be precisely determined from LQCD before the more complex ones.

\[ g_A^{QCD} = 1.2711(126) \]


\[ g_A^{UNCA} = 1.2772(020) \]

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Lattice QCD basics

Lattice QCD main idea: continuous 4D space \(\rightarrow\) hypercubical lattice.

Lattice spacing acts as a UV regulator

Quark Fields defined at Lattice points

Gluon Fields connect the Lattice points

Volume is finite

Discretize version for the lattice

\[
S = \int d^4x \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi} \left( \gamma_\mu \partial^\mu + m \right) \psi \right)
\]

Lattice Observables

\[
\langle O \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu e^{-S} \langle 0 | [A_\mu] \rangle
\]

\[
Z = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S}
\]

\[
\langle O \rangle = \frac{1}{N_I} \sum_i \langle 0 | U_i \rangle
\]
LQCD: Mesons and Baryons

\[ C(t) = \sum_k \langle \Omega | O_{\pi^+} | k \rangle \langle k | O_{\pi^+}^\dagger | \Omega \rangle e^{-E_k t} \]

\[ m_{\text{mesons}}^{\text{eff}}(t, \tau) = \frac{1}{\tau} \arccosh \left( \frac{C(t + \tau) + C(t - \tau)}{2C(t)} \right) \]

\[ \pi^+ a_t m_t = -0.0830 \quad a_t m_s = -0.0743 \quad a_t \delta = 0.0002 \]

\[ m_{\text{baryons}}^{\text{eff}}(t, \tau) = \frac{1}{\tau} \ln \left( \frac{C(t)}{C(t + \tau)} \right) \]

\[ \frac{\text{Signal}}{\text{Noise}} \sim \sqrt{N_{\text{constant}}} \]

Excited states contamination

\[ \frac{\text{Signal}}{\text{Noise}} \sim \sqrt{N} e^{(m_N - \frac{3}{2} m_{\pi^+}) t} \]

LQCD: Physics from numerical results

- Continuum and Infinite Volume extrapolation

\[ V \to \infty, \quad a \to 0 \]

- Physical pion mass limit
Nucleon Axial Coupling: Standard Method

- Fixed source-sink separation is fixed
- Excited states contributions drop in the long $t_{\text{sep}}$ limit:

$$R_3 = g_\lambda + z_1 e^{-t_{\text{sep}}\Delta_{10}} + z_{10} e^{-\tau - t_{\text{sep}}/2 \Delta_{10}}$$

$$t_{\text{sep}} \to \infty \quad R_3 \to \text{constant}$$

$t_{\text{sep}} = 10, 12, 14$

Smaller e.s contributions, Stat. error increases

Bhattacharya, Cirigliano, Cohen, Gupta, Lin, and Yoon 2016

*Phys. Rev. D94.5*
On the Feynman-Hellmann Theorem in Quantum Field Theory and the Calculation of Matrix Elements

Chris Bouchard, Chia Cheng Chang, Thorsten Kurth, Kostas Orginos, Andre Walker-Loud

(Submitted on 21 Dec 2016 (v1), last revised 5 Jul 2017 (this version, v2))

\[ \partial_\lambda E_\lambda^n = \langle n | H_\lambda | n \rangle \]

\[ C_\lambda(t) = \langle \lambda | \mathcal{O}(t)\mathcal{O}^\dagger(0) | \lambda \rangle = \frac{1}{Z} \int \mathcal{D}\Phi e^{-S - S_\lambda \mathcal{O}(t)\mathcal{O}^\dagger(0)} \]

\[ \frac{\partial m_{\text{eff}}}{\partial \lambda} \bigg|_{\lambda=0} = \frac{1}{\tau} \begin{Bmatrix} \frac{\partial_\lambda C(t)}{C(t)} - \frac{\partial_\lambda C(t+\tau)}{C(t+\tau)} \end{Bmatrix} \bigg|_{\lambda=0} \]

\[ \partial_\lambda C_\lambda(t) \quad \rightarrow \quad C(t) \quad \rightarrow \quad \int d^4x \quad \mathcal{J}(x) \]
Feynman-Hellman Method

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\[ \left. \frac{\partial m_{\text{eff}}}{\partial \lambda} \right|_{\lambda=0} = \frac{1}{\tau} \left\{ \frac{\partial_\lambda C(t)}{C(t)} - \frac{\partial_\lambda C(t + \tau)}{C(t + \tau)} \right\} \left|_{\lambda=0} \right. \]

Similar methods

Maiani, Martinelli, Paciello, and Taglienti 1987
Bulava, Donnellan, and Sommer 2012
Dvitiis, Petronzio, and Tantalo 2012
Chambers 2014
Savage, Shanahan, Tiburzi, Wagman, Winter, Beane, Chang, Davoudi, Detmold, and Orginos 2017
Correlator Analysis

Simultaneous fit to $C(t), \partial_\lambda m_{\text{eff}}^{g_A}, \partial_\lambda m_{\text{eff}}^{g_V}$ (SS and SP)

Two-state fit to ground and first excited state

Stability analysis for minimum and maximum time ranges
Standard vs Our Method
Renormalization

- Lattice calculation bare currents must be renormalized.

\[ g^r_A = Z_A g^b_A \]
\[ g^r_V = Z_V g^b_V = 1 \]

\[ g^r_A = \left( \frac{Z_A}{Z_V} \right) g^b_A \]

- Renormalization is done non-pertubatively with the RI/SMOM method and Mobius Domain Wall Fermions.

- \( \frac{Z_A}{Z_V} \approx 1 \) is consistent with good chiral symmetry properties for the Mobius Domain Wall action.
Renormalized Results

\[ m_\pi = \{130, 220, 310, 350, 400\} \text{ MeV} \]
Physical Results

Lattice Observables \[ a, V, m_\pi \] extrapolation

Continuum extrapolation

\[ g_A^{LQCD}(\epsilon_\pi^{\text{phys}}, \epsilon_a) \]

\[ g_A^{PDG} = 1.2723(23) \]

\[ \phi \]

Model average

\[ g_A^{(130), \epsilon_a} \]

\[ g_A^{(350), \epsilon_a} \]

\[ g_A^{(220), \epsilon_a} \]

\[ g_A^{(400), \epsilon_a} \]

\[ g_A^{(310), \epsilon_a} \]

\[ \epsilon_a^2 = a^2/(4\pi W_0^2) \]
\[ \delta_L = 8g_0^3 \epsilon_\pi^2 \sqrt{2\pi} \frac{e^{-m_\pi L}}{\sqrt{m_\pi L}} + O \left( e^{-\sqrt{2}m_\pi L}, \frac{1}{(m_\pi L)^{3/2}} \right) \]
\[ \delta_{L3} \equiv f_3 \epsilon_\pi^3 F_1(m_\pi L) \]
\[ \delta'_L \equiv \delta_L + \delta_{L3} \]
**Extrapolation Models**

NNLO $\chi$PT: \[ \text{Eq. (S8)} + \delta_a + \delta_L' \] \hfill (S21a)

NNLO+ct $\chi$PT: \[ \text{Eq. (S8)} + c_4 \epsilon_\pi^4 + \delta_a + \delta_L' \] \hfill (S21b)

NLO Taylor $\epsilon_\pi^2$: \[ c_0 + c_2 \epsilon_\pi^2 + c_4 \epsilon_\pi^4 + \delta_a + \delta_L' \] \hfill (S21c)

NNLO Taylor $\epsilon_\pi^2$: \[ c_0 + c_2 \epsilon_\pi^2 + c_4 \epsilon_\pi^4 + \delta_a + \delta_L' \] \hfill (S21d)

NLO Taylor $\epsilon_\pi$: \[ c_0 + c_1 \epsilon_\pi + \delta_a + \delta_L' \] \hfill (S21e)

NNLO Taylor $\epsilon_\pi$: \[ c_0 + c_1 \epsilon_\pi + c_2 \epsilon_\pi^2 + \delta_a + \delta_L' \] \hfill (S21f)
Analysis Details

Stability of Extrapolation Analysis

Final result

- 6 fits included in the model average and their relative weights
- discretization corrections
- finite volume corrections
- sensitivity to prior width
- sensitivity to pion mass cuts
- sensitivity to lattice spacing cuts
- additional XPT analysis
Analysis Details

Stability of Extrapolation Analysis

$m_\pi \leq 400$ MeV

$m_\pi \leq 350$ MeV

$m_\pi \leq 310$ MeV

2x LO width

2x all widths

$m_\pi \leq 350$ MeV

$m_\pi \leq 310$ MeV

$m_\pi \geq 220$ MeV

$a \leq 0.12$ fm

$a \geq 0.12$ fm

N3LO xPT

NLO xPT(\Delta)

$\xi = m_\pi/(4\pi F_\pi)$

sensitivity to prior width

sensitivity to pion mass cuts

sensitivity to lattice spacing cuts

additional XPT analysis

Slide from A. Walker-Loud
Analysis Details

Stability of Extrapolation Analysis

\[ a \simeq \{0.09, 0.12, 0.15\} \text{ fm} \quad \quad a \simeq \{0\times9, 0.12, 0.15\} \text{ fm} \quad \quad a \simeq \{0.09, 0.12, 0.15\} \text{ fm} \]

- Model average
- NNLO \( \chi^2 \) PT

Final result

- \( g_A^{LODD}(e^-, a = 0) \)
- \( g_A^{LODD} = 1.272(23) \)

- Sensitivity to lattice spacing cuts
- Additional XPT analysis

- Cuts
- \( m_{\pi} \leq 310 \text{ MeV} \)
- \( m_{\pi} > 220 \text{ MeV} \)
- \( a \leq 0.12 \text{ fm} \)
- \( a \geq 0.12 \text{ fm} \)
Final Result

\[ g_A = 1.2711^{+0.103}_{-0.039} \chi^{+0.15}_{-0.09} V^{+0.04}_{-0.01} I^{+0.05}_{-0.05} \]

Summary

- Feynman-Hellman method:
  - computationally cheaper
  - enables access to early $t_{sep}$ where signal-to-noise problem is less significant

- Percent level determination of nucleon axial charge $g_A^{QCD} = 1.2711(126)$ and getting closer to experimental errors

- Improve results coming up in next talk!
Team

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Thank you!