Pion-pion scattering and the timelike pion form factor from Lattice QCD

Ben Hörz (LBNL)

MENU 2019

Carnegie Mellon University, Pittsburgh

June 6, 2019

Hadron interactions from Lattice QCD

[NuSTEC White Paper: Status and Challenges of Neutrino-Nucleus Scattering 1706.03621]

However better knowledge of contributions from heavier resonances is also important for higher energy experiments like NOvA and DUNE and seriously lacking.

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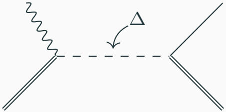
- improving our knowledge of the axial part of nucleon-Δ transition matrix elements, either via a new hydrogen and/or deuterium experiment or via lattice-QCD calculations;
- describing nonresonant contributions to pion production channels. Understanding the range



$\langle N\pi | J_\mu | N \rangle$

multi-hadron state

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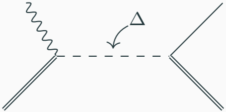

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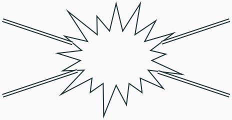

nucleon-nucleon interactions

nucleon-hyperon interactions

$N\Sigma, N\Lambda$

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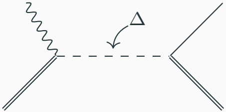

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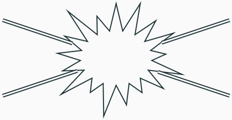

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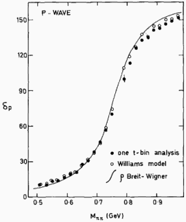
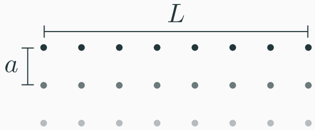
[Estabrooks, Martin 1975]



Salient Features of Lattice QCD



- lattice spacing a
- spatial extent L
- Euclidean time

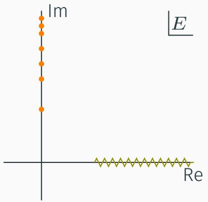
basic observable: correlation functions

'$\pi\pi$' creation operator

$$C_{ij}(t) = \langle 0|O_i(t)\ \bar{O}_j(0)|0\rangle$$

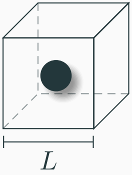
$$= \sum_n \langle 0|O_i|n\rangle\ \langle n|\bar{O}_j|0\rangle\ \mathrm{e}^{-E_n t}$$

discrete spectrum



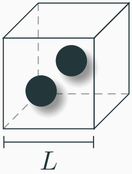
How do we get scattering from finite-volume observables?

[see also talk by M. Mai earlier this week]



single particle in a periodic box
$\rightsquigarrow \Delta E \propto \mathrm{e}^{-mL}$

two spinless particles in a periodic box
$\rightsquigarrow \Delta E \propto a_0/L^3 + O(L^{-4})$ [Lüscher '86, '91]



$\Rightarrow$ 'The Lüscher method'

Scattering processes and resonances from lattice QCD

Raúl A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] and Ross D. Young[3,‡]

[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

[2] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*

[3] *Special Research Center for the Subatomic Structure of Matter (CSSM), Department of Physics, University of Adelaide, Adelaide 5005, Australia*

(Dated: June 21, 2017)

The vast majority of hadrons observed in nature are not stable under the strong interaction, rather they are *resonances* whose existence is deduced from enhancements in the energy dependence of scattering amplitudes. The study of hadron resonances of-

[Briceño, Dudek, Young 1706.06223]

Two-particle Quantization Condition

2-particle channel
partial wave
(total angular mom.)

$$\det\left[\mathcal{M}^{-1}(E_L) + F(E_L, L)\right] = 0$$

E_L – FV spectrum
$\mathcal{M}$ – 2-to-2 scatt. ampl.
F – known functions

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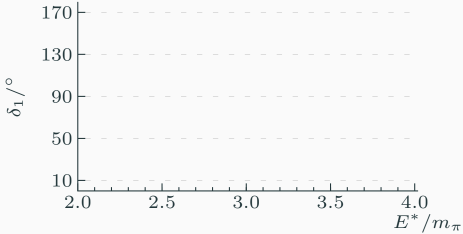
F – known functions

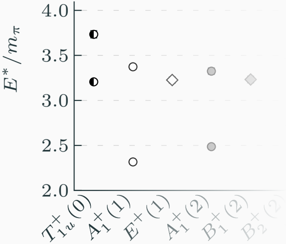
group theory worked out and publicly available on Github

[Morningstar, Bulava, Singha, Brett, Fallica, Hanlon, BH 1707.05817]

A simple (yet relevant) resonance: $\rho(770)$

[plot adapted from Bulava, Fahy, BH, Juge, Morningstar, Wong 1604.05593]





- elastic $\pi\pi$ scattering neglecting $\ell \geq 3$ partial wave
  spectrum ⇔ scattering amplitude
- benchmark system for the lattice

e.g. Lang et al. 1105.5636, Aoki et al. 1106.5365, ..., Dudek et al. 1212.0830, ...

- recent interest due to its contribution to $(g-2)_\mu$ HVP

[Meyer, Wittig 1807.09370]

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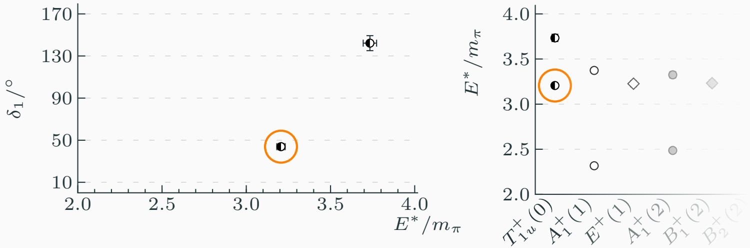

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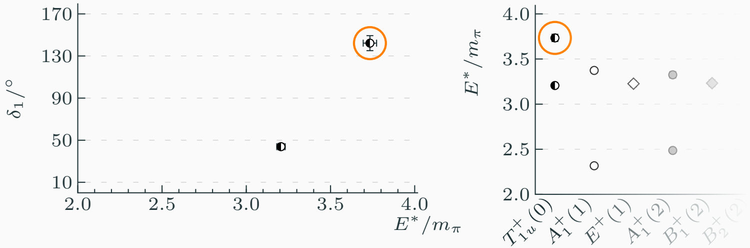

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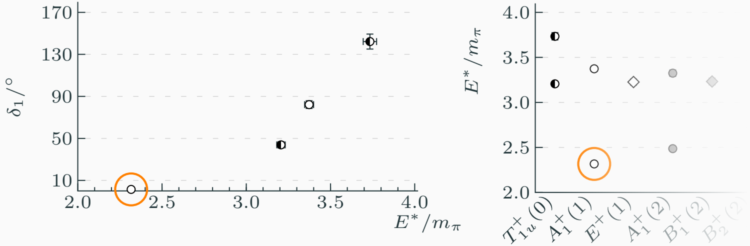

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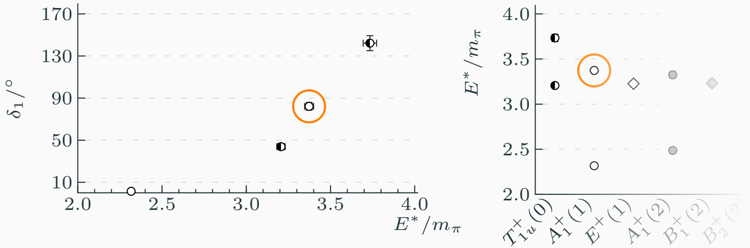

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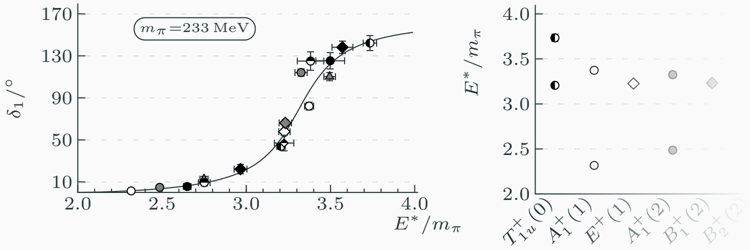

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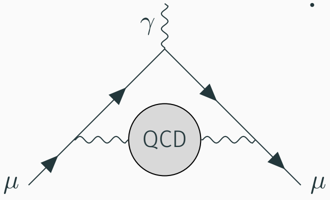
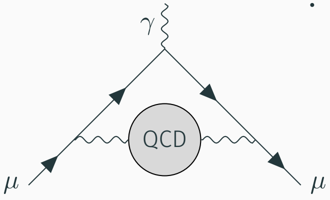


- muon anomalous magnetic moment $(g-2)_\mu$
  - HVP governed by
    $R_{\text{had}} = \sigma(e^+e^- \to \text{hadrons}) / \frac{4\pi\alpha_{\text{em}}(s)^2}{3s}$
  - two-pion state dominates at low energies
    $R_{\text{had}}(s) = \frac{1}{4}\left(1 - \frac{4m_\pi^2}{s}\right)^{\frac{3}{2}} |F_\pi(s)|^2$
    $\gamma^* \to \pi\pi$

Matrix elements: Timelike pion form factor



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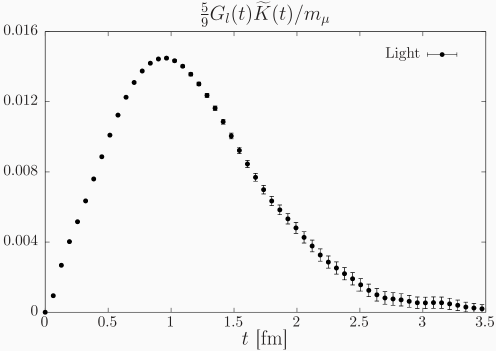
infinite volume — finite volume

$$|F_\pi(E^*)|^2 = g_\Lambda(\gamma)\, q \frac{\partial(\delta_1 + F)}{\partial q} \frac{3\pi E^{*2}}{2q^5 L^3} \left|\langle 0|V^{(\mathbf{d},\Lambda)}|\mathbf{d}\Lambda E^*\rangle\right|^2$$

requires scattering amplitude

Lellouch, Lüscher hep-lat/0003023
Meyer 1105.1892
Feng et al. 1412.6319

Vector-correlator reconstruction



$m_\pi = 200\,\mathrm{MeV}, L = 4.1\,\mathrm{fm}$

[Gérardin, Cè, von Hippel, BH, Meyer, Mohler, Ottnad, Wilhelm, Wittig 1904.03120]

- time-momentum representation
  [Bernecker, Meyer 1107.4388]
- clear spectral decomposition

$$G_l(t) \sim \sum_{\mathbf{x}} \langle 0|V(\mathbf{x},t)V^\dagger(0)|0\rangle$$
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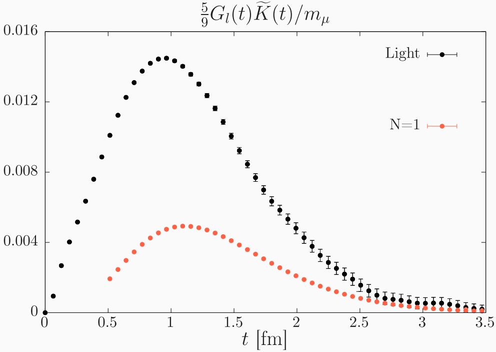

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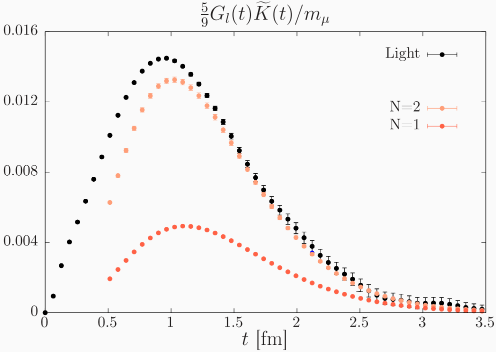

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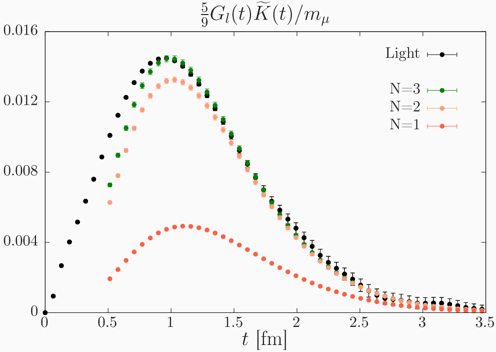

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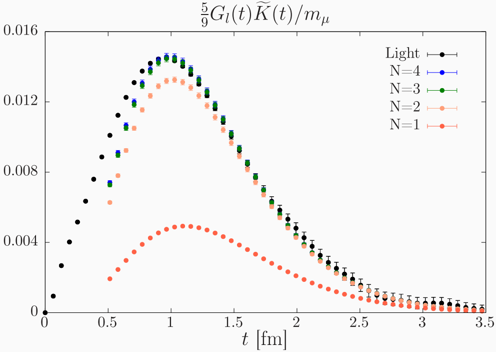

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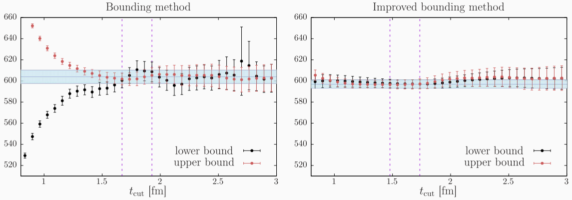
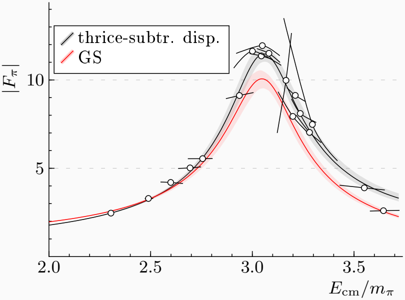
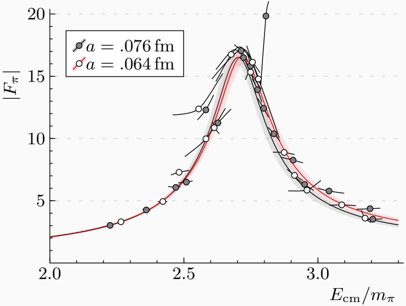


[Andersen, Bulava, BH, Morningstar 1808.05007]



$m_\pi = 260\,\mathrm{MeV}$



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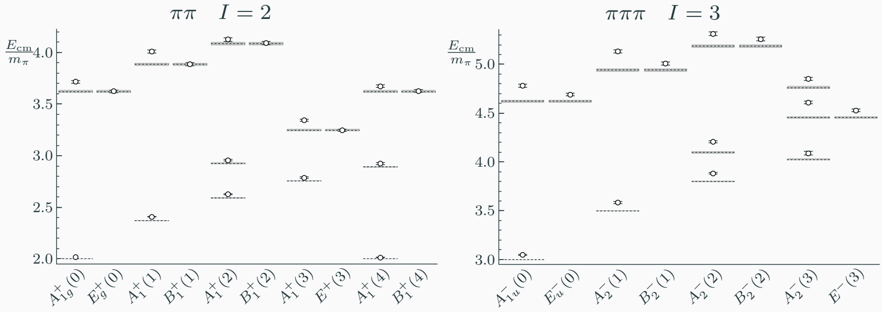
⇝ formalism and technology under control for two-meson systems



$m_\pi = 200\,\mathrm{MeV}, \quad m_\pi L = 4.2$ [BH, Hanlon 1905.04277]

- significant energy shifts in three-pion ground and excited states

  (remember: energy shifts encode interaction)

- finite-volume formalism to reveal three-particle interaction

  much development in recent years [Hansen, Sharpe 1901.00483]

  Sharpe, Hansen, Briceño, Hammer, Rusetsky, Polejaeva, Mai, Döring, …

Summary & Outlook

- hadron interactions from lattice QCD ...

  (calculations maturing, starting to assess standard lattice systematics)

- ...and with practical relevance

  (helping to improve $(g-2)_\mu$/HVP from lattice QCD)

- similar method for access to Δ transition form factor

  (for $N\pi$ scattering results see talk by J. Bulava)

- stay tuned for
  - NN scattering
  - $N\Sigma$, $N\Lambda$ scattering
  - ...